



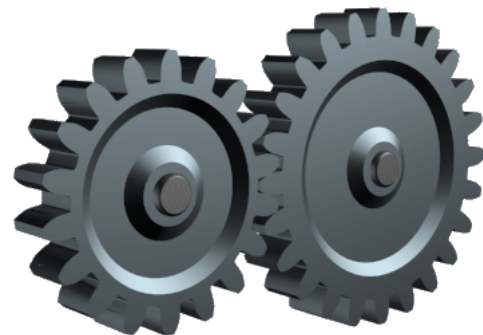
第03章

齿轮传动设计

第01节 齿轮的机构

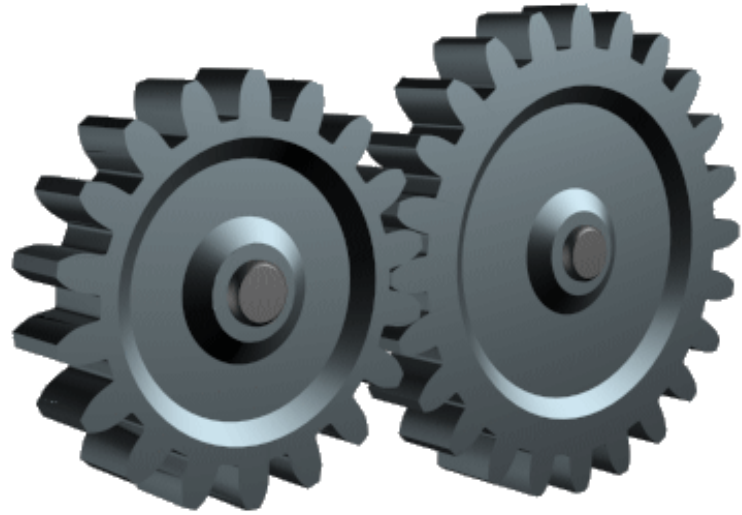
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本章要点概述

- 齿轮机构及其设计
 - 齿轮机构的类型与特点
 - 渐开线直齿圆柱齿轮机构
 - 其他齿轮机构及其传动分析
- 齿轮传动设计
 - 齿轮传动的失效形式和设计约束
 - 直齿圆柱齿轮传动的强度条件
 - 齿轮材料和许用应力
 - 斜齿圆柱齿轮传动的强度条件
 - 直齿锥齿轮传动的强度条件
 - 齿轮传动的设计方法
 - 行星齿轮传动设计概要
 - 曲线齿锥齿轮和准双曲面齿轮传动
 - 齿轮传动类型的选择
- 蜗杆传动设计
 - 圆柱蜗杆传动主要参数及几何尺寸
 - 蜗杆传动的失效形式和设计约束
 - 圆柱蜗杆传动的强度条件
 - 蜗杆传动的效率和热平衡计算
 - 圆柱蜗杆传动的设计方法
 - 环面蜗杆传动
 - 蜗杆传动类型的选择
- 齿轮系及其设计
 - 定轴齿轮系及其传动比
 - 周转齿轮系及其传动比
 - 复合齿轮系及其传动比
 - 齿轮系的应用
 - 行星轮系设计
 - 其他行星传动简介



齿轮机构及其设计

齿轮机构的类型与特点

渐开线直齿圆柱齿轮机构

其他齿轮机构及其传动分析

齿轮传动是机械传动中最重要、应用最广泛的一种传动方式

优点

- 传动效率高
- 工作可靠
- 寿命长
- 传动比准确
- 结构紧凑

缺点

- 制造精度要求高
- 制造费用大
- 精度低时振动和噪声大
- 不宜用于轴间距离较大的传动

齿轮传动

- 开式齿轮传动
 - 齿轮完全外露，易落入灰砂和杂物，不能保证良好的润滑，故轮齿易磨损，该传动方式多用于低速、不重要的场合
- 闭式齿轮传动
 - 其齿轮和轴承完全封闭在箱体内，能保证良好的润滑和较好的啮合精度，因此闭式齿轮传动应用较广泛

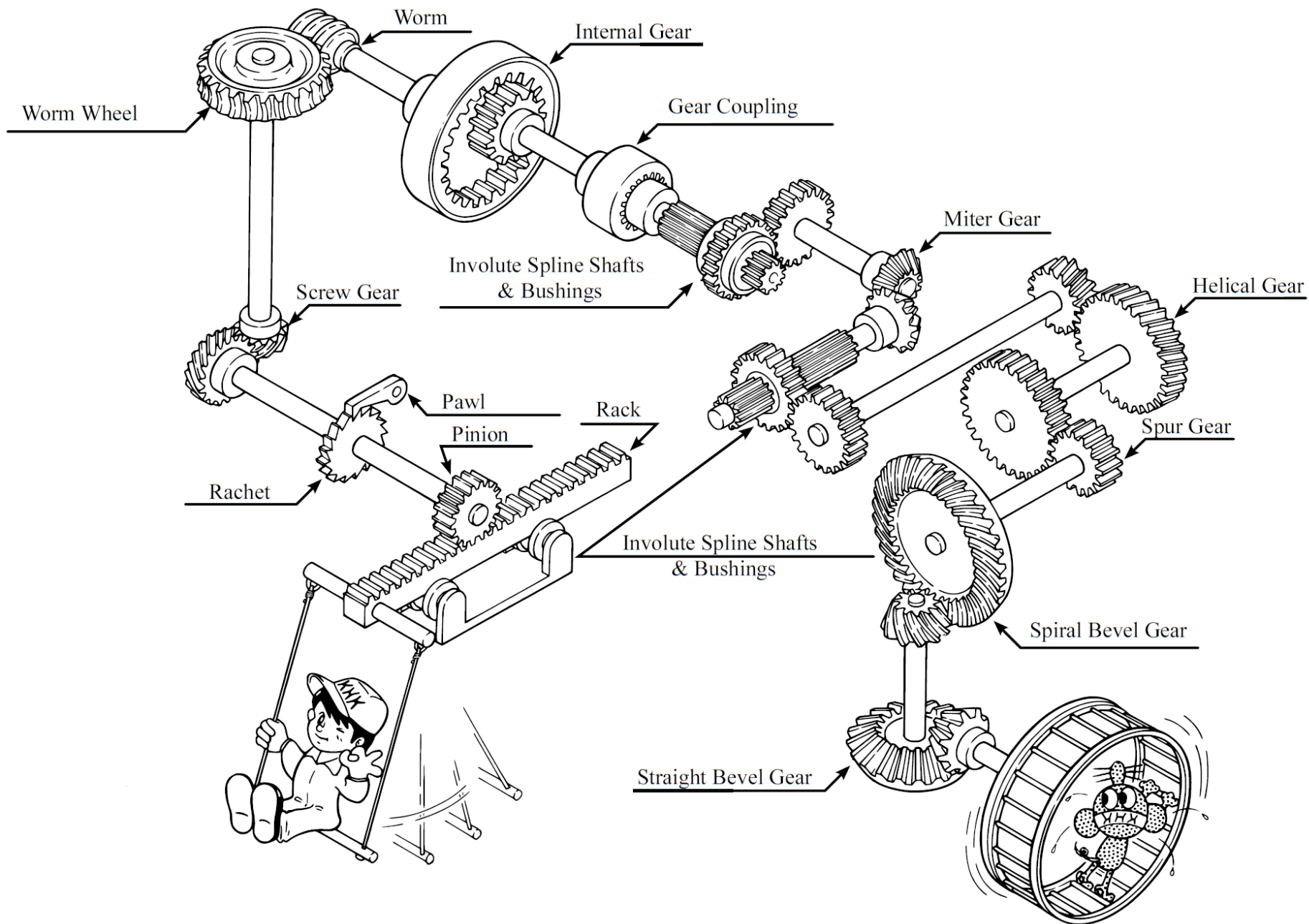
机械系统对齿轮传动的功能要求

- ① 能传递两个平行轴或相交轴或交错轴间的回转运动和转矩
- ② 能保证传动比恒定不变
- ③ 能传递足够大的动力，工作可靠
- ④ 能保证较高的运动精度
- ⑤ 能达到预定的工作寿命



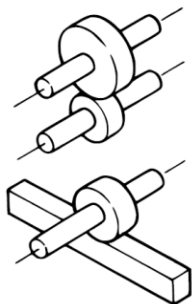
齿轮机构的 类型与特点

齿轮机构及其设计

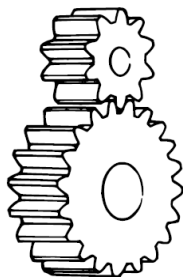


齿轮的类型

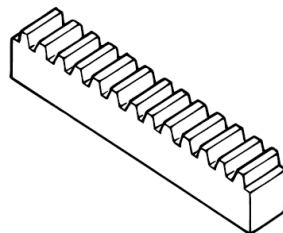
Parallel Axes



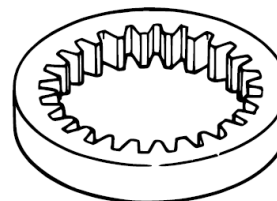
Spur Gear



Rack



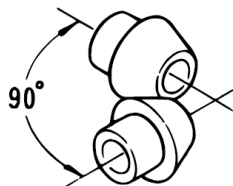
Internal Gear



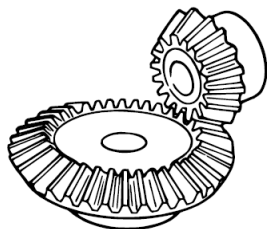
Helical Gear



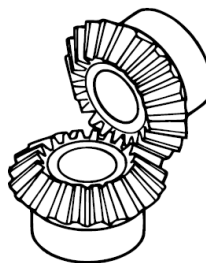
Intersecting Axes



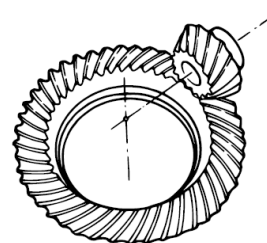
Bevel Gear



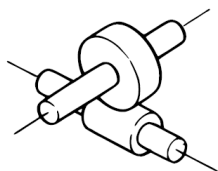
Miter Gear



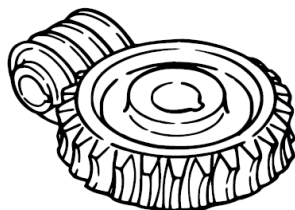
Spiral Bevel Gear



Nonparallel,
Nonintersecting
Axes



Worm & Worm Wheel

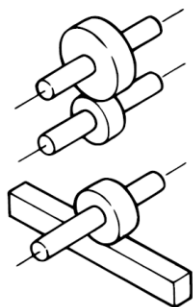


Screw Gear



齿轮的类型

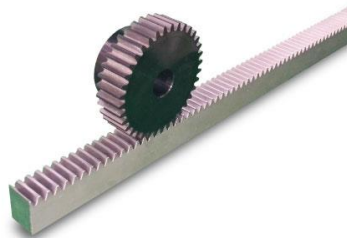
Parallel Axes



Spur Gear



Rack



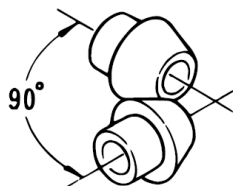
Internal Gear



Helical Gear



Intersecting Axes



Bevel Gear



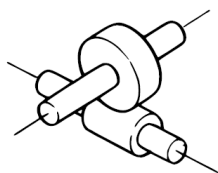
Miter Gear



Spiral Bevel Gear



Nonparallel, Nonintersecting Axes
















Worm & Worm Wheel



Screw Gear








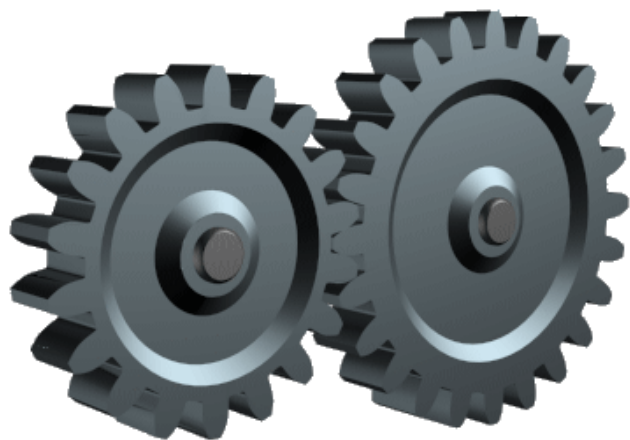
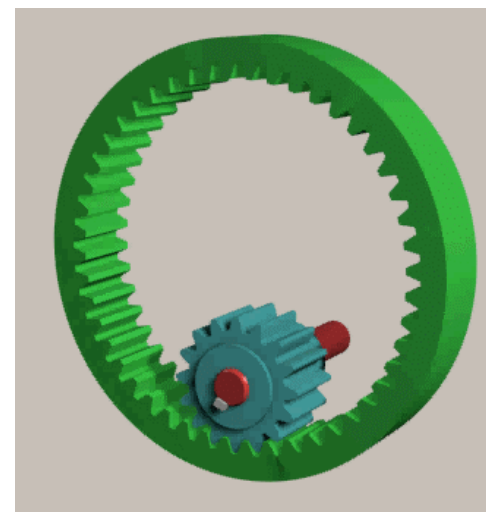
齿轮机构的类型

两轴线平行的圆柱齿轮机构					齿轮-齿条
外啮合			内啮合		直齿
直齿	斜齿	人字齿	直齿	斜齿	
					
					斜齿
两轴线相交的锥齿轮机构			两轴线交错的齿轮机构		
直齿	曲齿	交错轴斜齿轮	蜗杆传动	准双曲面齿轮	
					螺旋齿
					

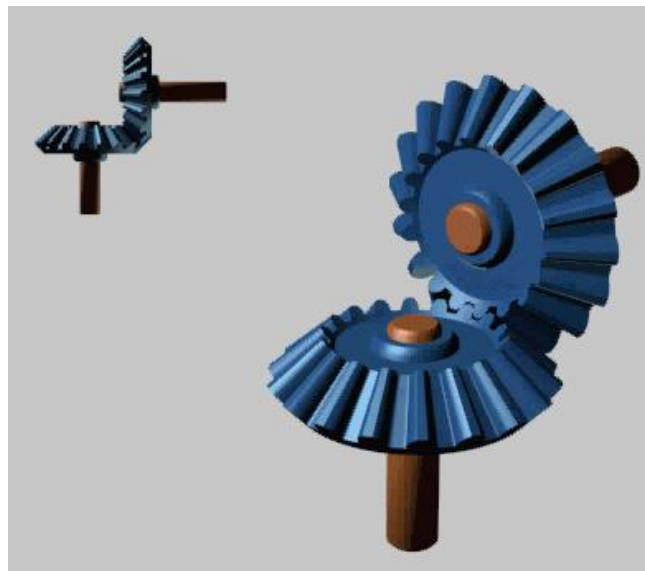
齿轮机构的类型


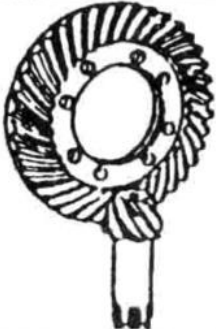
两轴线平行的圆柱齿轮机构

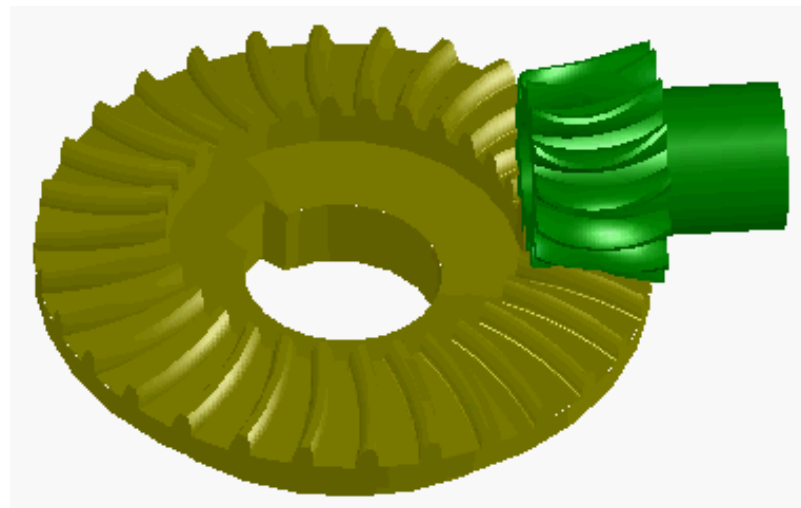
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直 齿	斜 齿	人 字 齿	直 齿	斜 齿
				



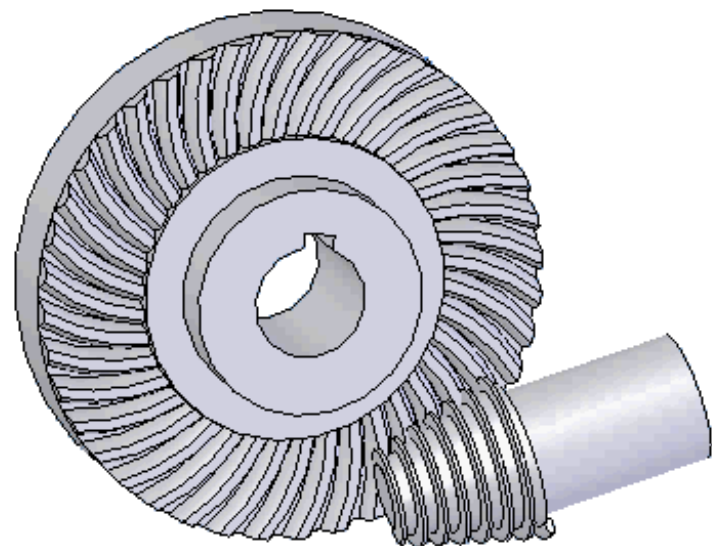
齿轮机构的类型

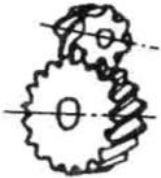

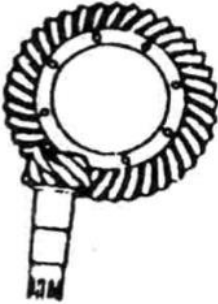


两轴线相交的锥齿轮机构	
直 齿	曲 齿
	

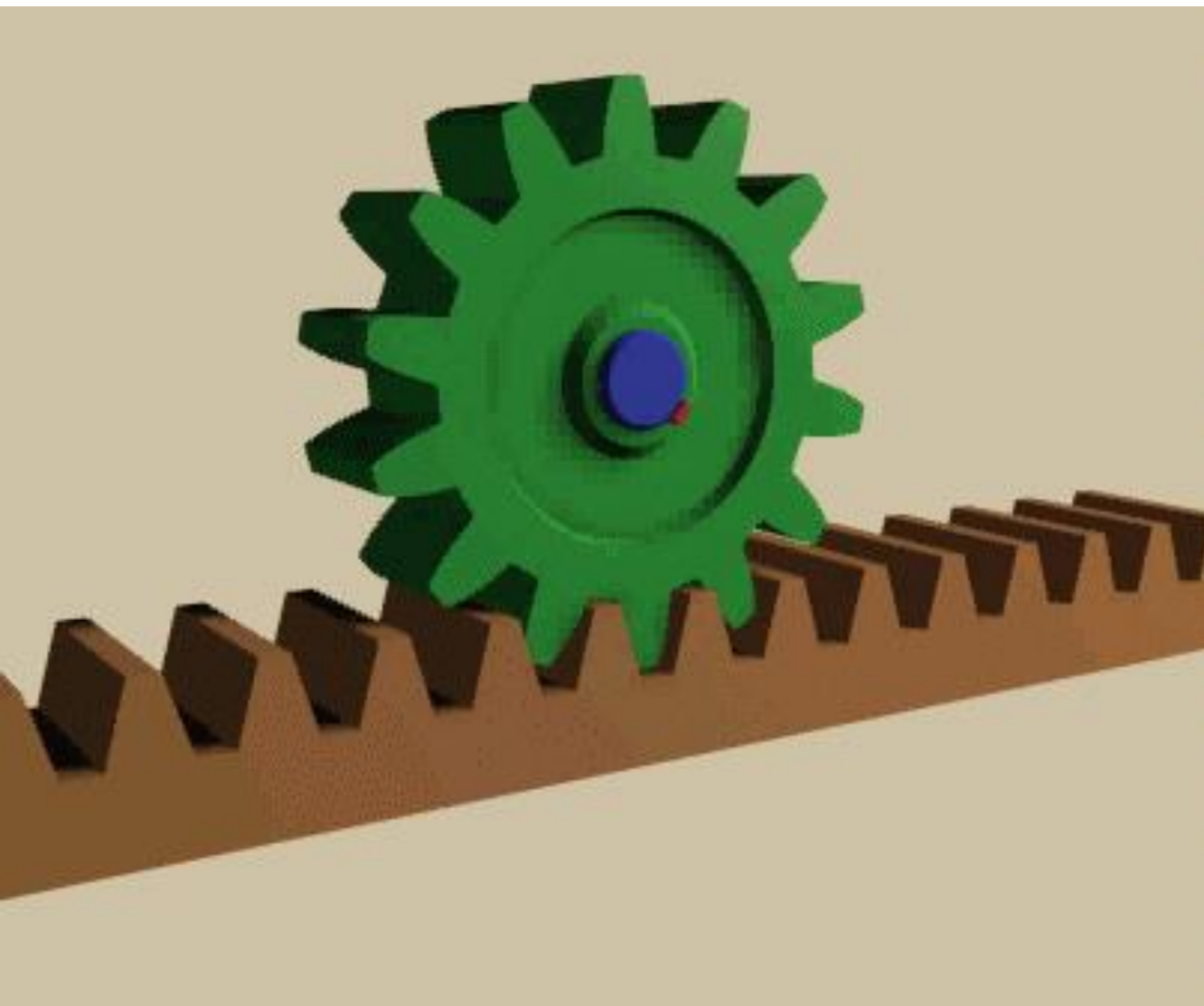


齿轮机构的类型



两轴线交错的齿轮机构		
交错轴斜齿轮	蜗杆传动	准双曲面齿轮
		

齿轮机构的类型



齿轮-齿条

直 齿



斜 齿

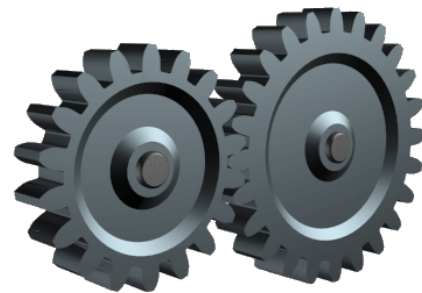


螺旋齿



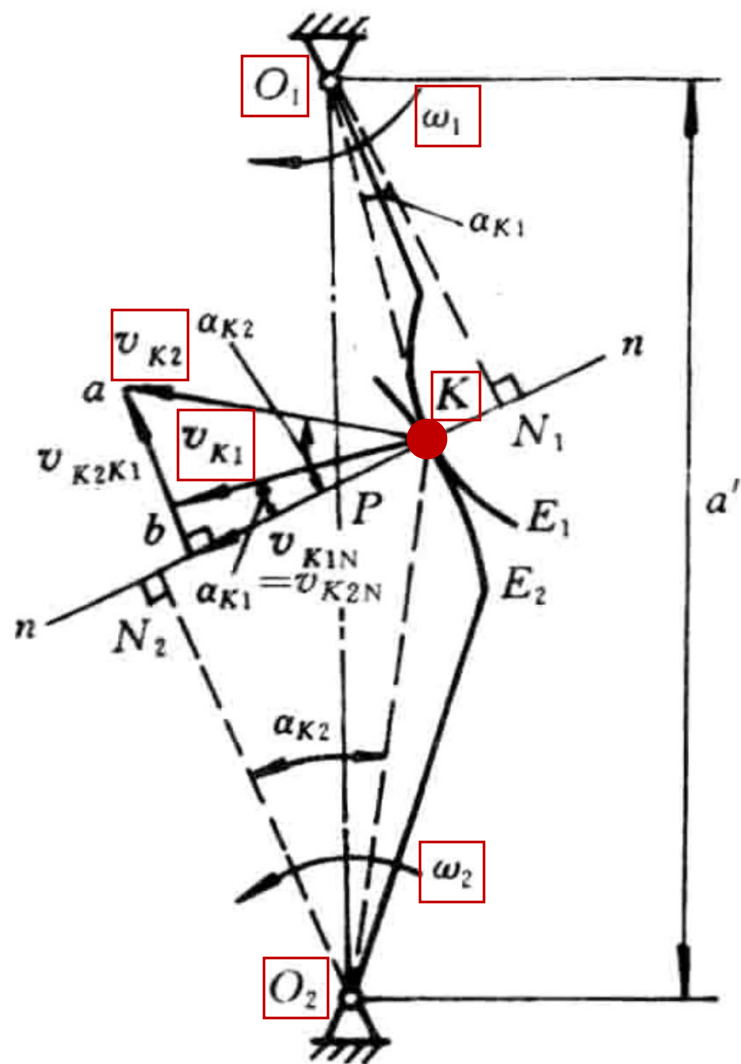
齿廓啮合基本定律

- 一对齿轮传递转矩和运动的过程，是通过这对齿轮主动轮上的齿廓与从动轮上的齿廓依次相互接触来实现的
 - 其瞬时传动比的变化规律与两轮齿廓曲线的形状（简称齿廓形状）有关
 - 齿廓形状不同，两轮瞬时传动比的变化规律也不同
 - 对齿轮传动的基本要求是传动准确、平稳
 - 即要求在传动过程中，瞬时传动比保持不变
- 当主动轮以等角速度 ω_1 回转时，一般要求从动轮以某一等角速度 ω_2 回转，否则将产生加速度和惯性力
 - 影响齿轮的寿命
 - 引起机器的振动和噪声，影响工作精度



齿廓形状需要满足瞬时传动比保持不变

- 瞬时传动比
 - 主、从动轮瞬时角速度的比值
 - $$i_{12} = \frac{\omega_1}{\omega_2}$$
- 如图所示为一对相啮合的齿轮齿廓
 - O_1 、 O_2 为两轮的转动中心
 - E_1 、 E_2 为两轮相互啮合的对齿廓
 - 若两齿廓某一瞬时在任意点 K 接触
 - 齿轮 1 为主动件，以瞬时角速度 ω_1 绕 O_1 顺时针转动，
 - 带动齿轮 2 以瞬时角速度 ω_2 绕 O_2 逆时针转动
 - 则在此瞬时两轮在点 K 的速度分别为
 - $v_{K1} = \omega_1 \overline{O_1K}$, $v_{K2} = \omega_2 \overline{O_2K}$

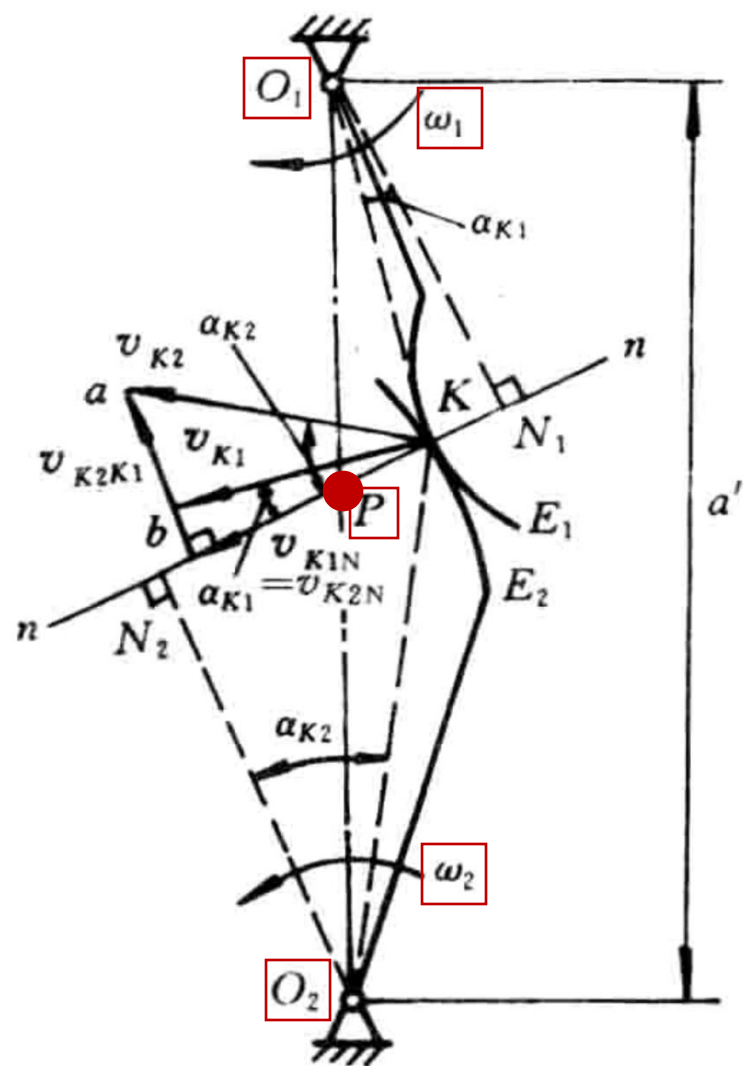


齿廓形状需要满足瞬时传动比保持不变

- 由于两轮的齿廓是刚体，且连续接触，故其速度 v_{K1} 和 v_{K2} 在公法线上的速度分量应相等，即
 - $v_{K1N} = v_{K2N}$
 - 其相对速度的方向应垂直于齿廓接触处的公法线 $n-n$ (即 $ab \perp n-n$)

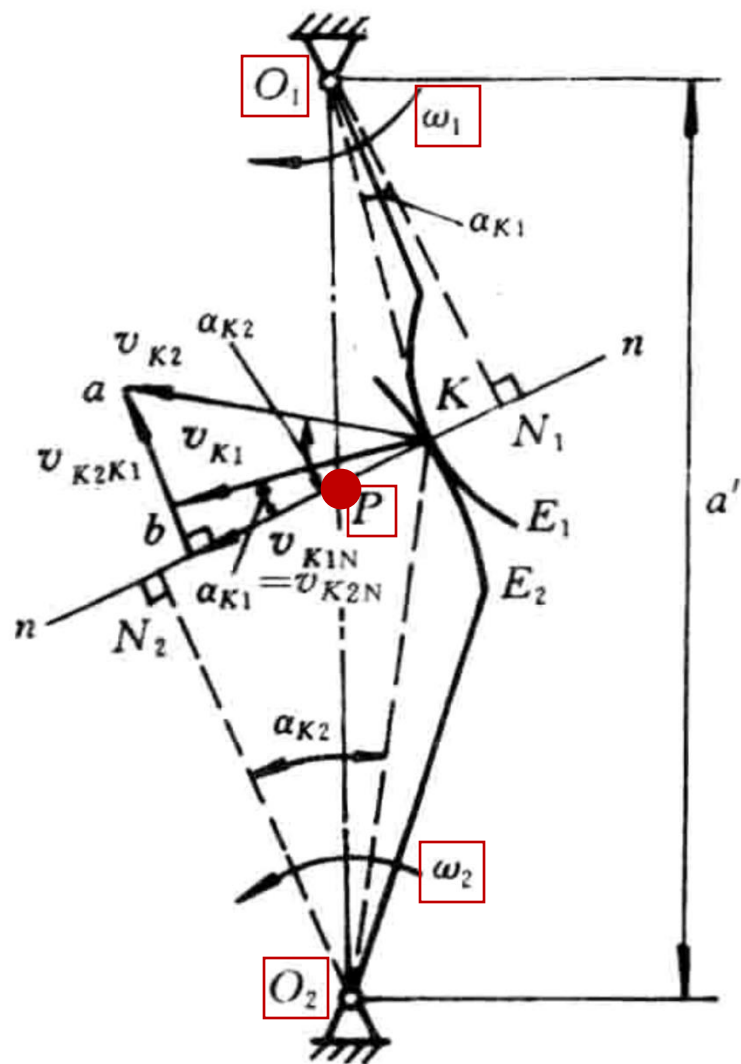
- 过点 K 作两齿廓的公法线 $n-n$ 交连心线 O_1O_2 于点 P
 - 由理论力学中的三心定理可知，点 P 是两齿轮的相对瞬心
 - 根据瞬心是两构件上相对速度为零的重合点，即瞬心也是两构件在该瞬时具有相同绝对速度的重合点
 - 两构件在任一瞬时的相对运动都可看成绕该瞬心的相对转动，故有

$$v_P = \omega_1 \overline{O_1P} = \omega_2 \overline{O_2P} \Rightarrow i_{12} = \frac{\omega_1}{\omega_2} = \frac{\overline{O_2P}}{\overline{O_1P}}$$



齿廓形状需要满足瞬时传动比保持不变

- $v_P = \omega_1 \overline{O_1P} = \omega_2 \overline{O_2P} \Rightarrow i_{12} = \frac{\omega_1}{\omega_2} = \frac{\overline{O_2P}}{\overline{O_1P}}$
- 要使两轮的瞬时传动比恒定不变，比值 $\overline{O_2P}/\overline{O_1P}$ 应为常数
 - 因两轮中心距 O_1O_2 为定长，若要满足上述要求，则必须使点 P 为连心线上的一个固定点
- 此固定点 P 称为节点
 - 分别以 O_1 和 O_2 为圆心，可作出过节点 P 的两个相切的圆，称为节圆，其半径用 r'_1 和 r'_2 表示
- 由此得 **齿廓啮合的基本定律**
 - 要使两齿轮传动的瞬时传动比为一常数，必须满足不论两齿廓在任何位置接触，过接触点所作的齿廓公法线与连心线应相交于一固定点的条件



齿廓形状需要满足瞬时传动比保持不变

- 由齿廓啮合的基本定律，可得出如下结论

(1) 理论上，凡能满足齿廓啮合基本定律的一对齿廓（称为共轭齿廓）曲线，均可作为齿轮机构的齿廓，并能实现瞬时传动比恒定不变的要求

(2) 一对齿轮的传动过程，可以看做是其对节圆在作纯滚动，因而其外啮合中心距恒等于其节圆半径之和

(3) 只有当一对齿轮相互啮合传动时才存在节圆，单个齿轮不存在节圆

(4) 变传动比齿轮机构的节点P不再是一个定点，而是按一定规律在连心线上移动，节点P在两轮转动平面上的轨迹不是两个圆，而是两条封闭曲线（如在椭圆齿轮传动机构中，这两条封闭曲线是两个椭圆），一般称该封闭曲线为节线

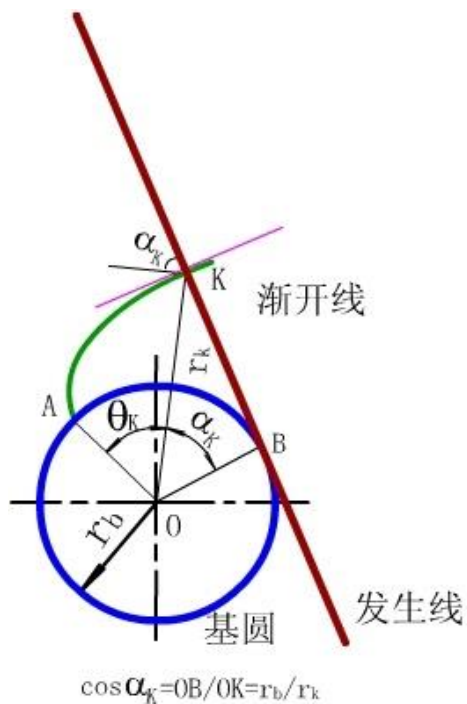
实际上，可作为共轭齿廓的曲线有无限多条，只要给定一个齿轮的齿廓曲线，就可以根据啮合基本定律，求出与其共轭的另一根齿廓曲线

但是，齿廓曲线的选择，除了应满足瞬时传动比恒定不变外，还应考虑制造、安装和强度等要求

在齿轮机构中，通常采用渐开线、摆线和圆弧等作为齿轮的齿廓曲线

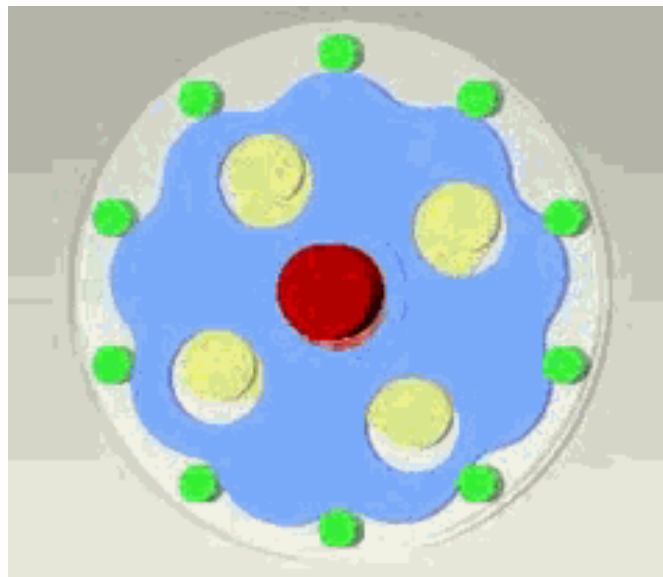
- 其中以渐开线齿廓应用最广

共轭齿廓

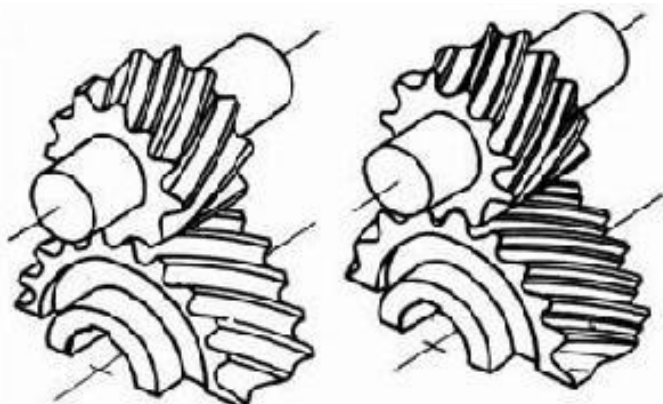


渐开线齿廓

摆线齿廓



圆弧齿廓

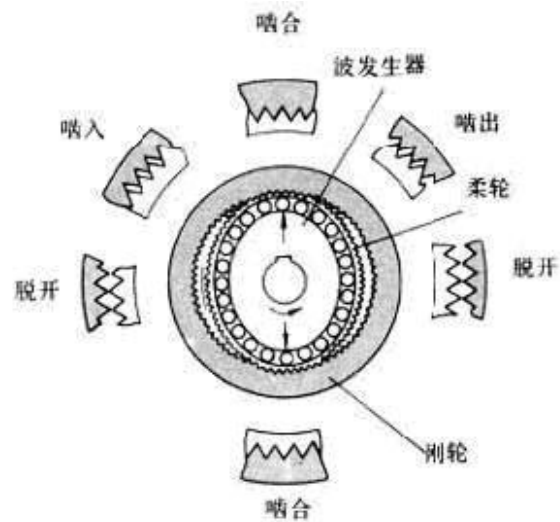


单圆弧齿轮传动

双圆弧齿轮传动

图1 圆弧齿轮啮合的两种形式

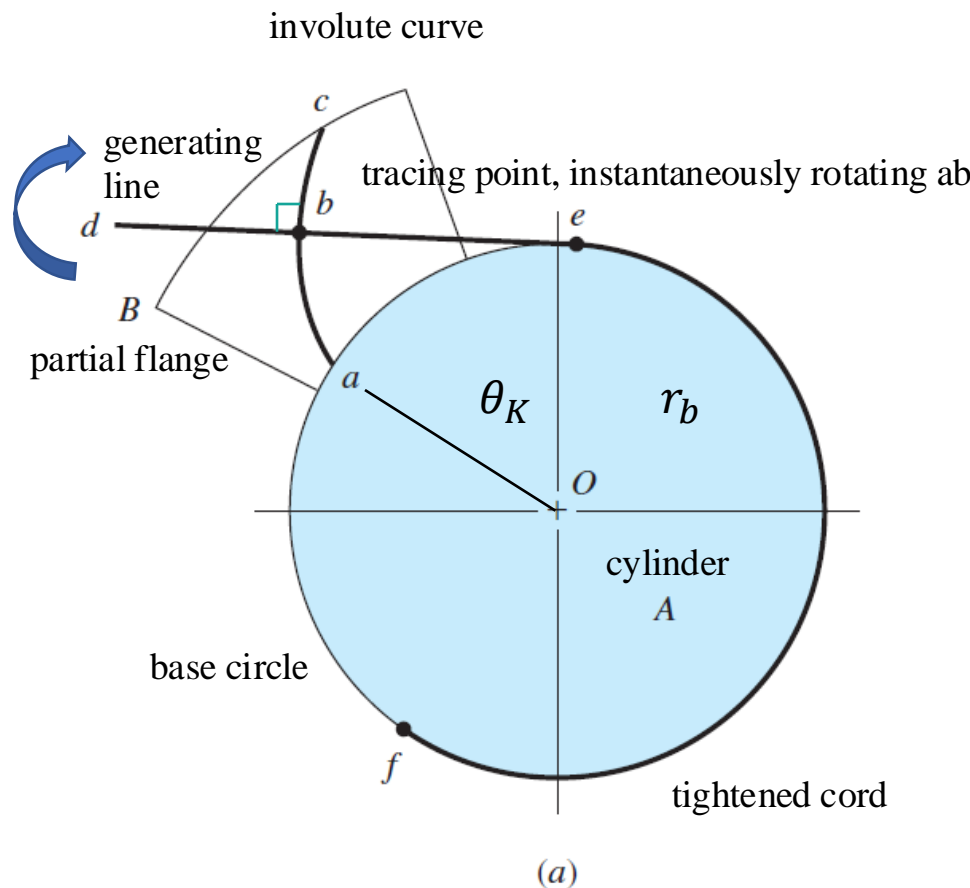
谐波齿廓



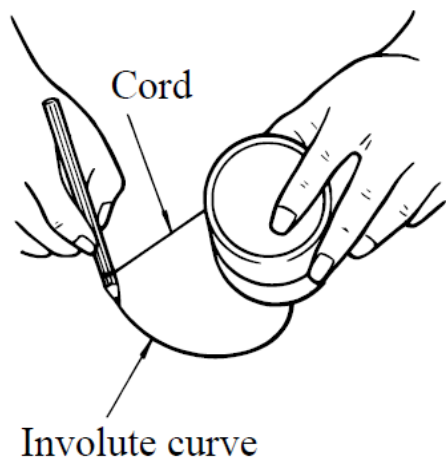
渐开线齿廓：渐开线的形成

• 如图所示，当直线沿一圆周作相切纯滚动时，直线上任一点在与该圆固连的平面之上的相应轨迹，称为该圆的渐开线

- 这个圆称为渐开线的基圆，其半径用 r_b 表示
- 该直线称为渐开线的发生线
- 角 θ_K 称为渐开线的展角

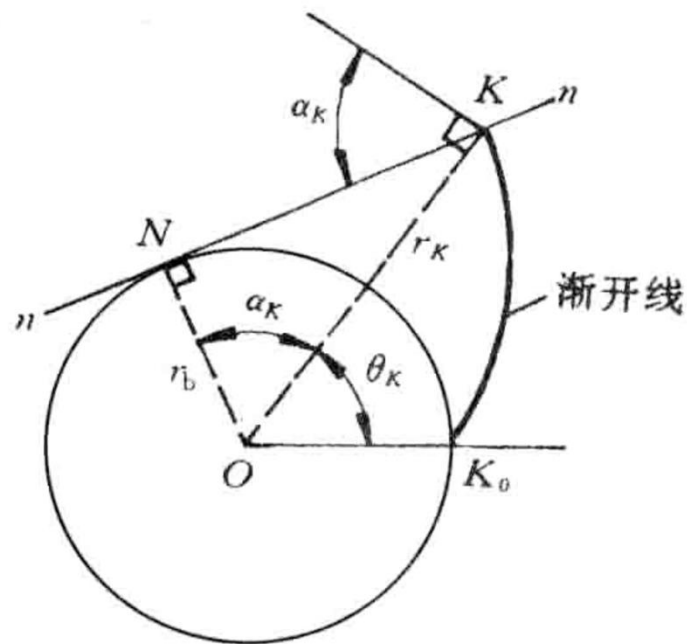


Generation of an Involute



渐开线齿廓：渐开线齿廓的性质

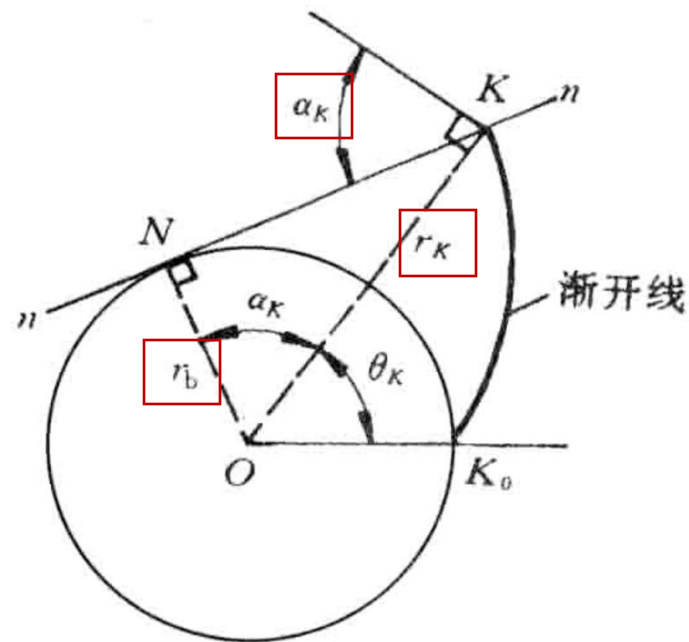
- (1) 发生线沿基圆滚过的长度等于基圆上被滚过的弧长，即 $\overline{NK} = \widehat{NK_0}$
- (2) 渐开线上任一点的法线必切于基圆，切于基圆的直线必为渐开线上某一点的法线
- (3) 如图所示，发生线与基圆的切点 N 是渐开线在点 K 的曲率中心，线段 NK 是渐开线在点 K 处的曲率半径（用 ρ_K 表示）。渐开线上愈接近基圆的点，其曲率半径愈小，在基圆上的点 K_0 ，其曲率半径等于零
- (4) 渐开线的形状取决于基圆的大小
 - 如图所示，渐开线上的任一点 K ，其曲率半径 $\rho_K = \overline{NK} = \sqrt{r_K^2 - r_b^2}$ ，因此，基圆愈大，点 K 的曲率半径就愈大，渐开线就愈平直；当基圆直径无穷大时，渐开线成为斜直线
 - 齿条的齿廓曲线就是由直线形成的渐开线
- (5) 基圆内无渐开线



渐开线齿廓：渐开线齿廓的性质

- (6) 在不考虑摩擦力、重力和惯性力的条件下，一对齿廓相互啮合时，轮齿在接触点 K 所受的正压力方向（沿法线 $n-n$ 的方向）与受力点线速度方向（齿轮绕轴心 O 转动时，齿廓上点 K 的线速度与 OK 垂直）之间所夹的锐角，称为齿轮齿廓在该点的压力角
 - 图中的 α_K 就是渐开线上点 K 的压力角： $\cos \alpha_K = r_b/r_K$
 - r_b 为渐开线的基圆半径； r_K 为渐开线上点 K 的向径

- 所以，渐开线齿廓上各点具有不同的压力角
 - 点 K 离圆心 O 愈远（即 r_K 愈大），其压力角也愈大
 - 当 $r_K = r_b$ 时， $\alpha_K = 0$ ，即渐开线在基圆上的压力角等于零



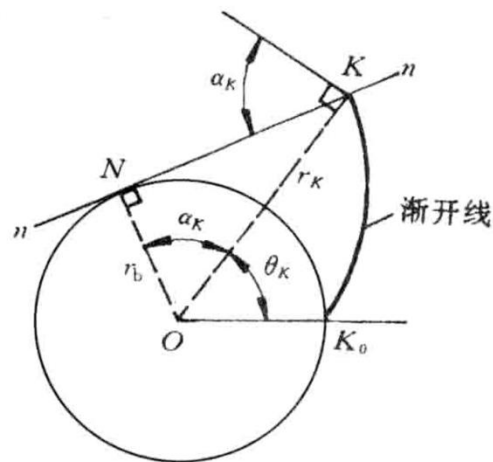
渐开线齿廓：渐开线的方程式

- 根据渐开线的性质，可导出渐开线的极坐标方程式

- 如图所示，以基圆中心为极坐标的原点，以渐开线起点 K_0 的向径 OK_0 为极坐标轴，则渐开线上任意点 K 的极坐标可用展角 θ_K 和向径 r_K 确定

$$\bullet \tan \alpha_K = \frac{\overline{NK}}{\overline{ON}} = \frac{\widehat{NK_0}}{r_b} = \frac{r_b(\alpha_K + \theta_K)}{r_b} \approx \alpha_K + \theta_K$$

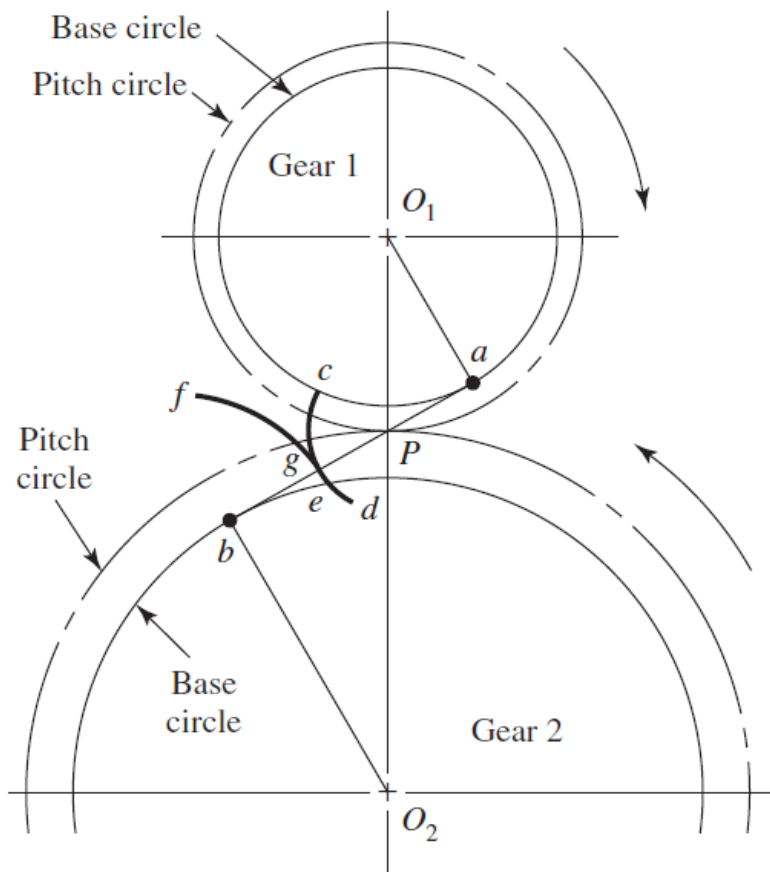
$$\bullet \theta_K = \tan \alpha_K - \alpha_K$$



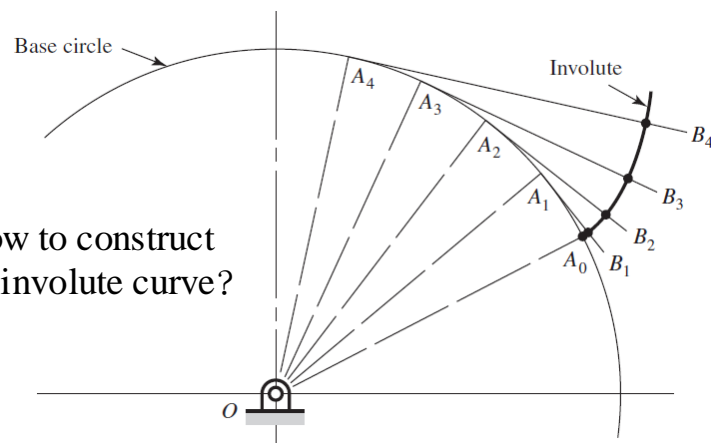
- 由上式可知，展角 θ_K 是随压力角 α_K 的大小而变化的，只要知道渐开线上某点的压力角 α_K ，则该点的展角 θ_K 便可由上式求出，即展角 θ_K 为压力角 α_K 的渐开线函数，用 $\text{inv } \alpha_K$ 表示，即有 $\theta_K = \text{inv } \alpha_K = \tan \alpha_K - \alpha_K$

- 可得渐开线的极坐标参数方程为
$$\begin{cases} r_K = r_b / \cos \alpha_K \\ \theta_K = \text{inv } \alpha_K = \tan \alpha_K - \alpha_K \end{cases}$$

渐开线齿廓：渐开线的绘制



(b)
Involute Action



1. Divide the base circle into a number of equal parts
2. Construct radial lines OA_0, OA_1, OA_2 , etc.
3. Beginning at A_1 , construct perpendiculars A_1B_1, A_2B_2, A_3B_3 , etc.
4. Along A_1B_1 lay off the distance A_1A_0 ,
5. Along A_2B_2 lay off **twice** the distance A_1A_0 , etc.,
6. Producing points through which the involute curve can be constructed.

渐开线齿廓：渐开线的绘制

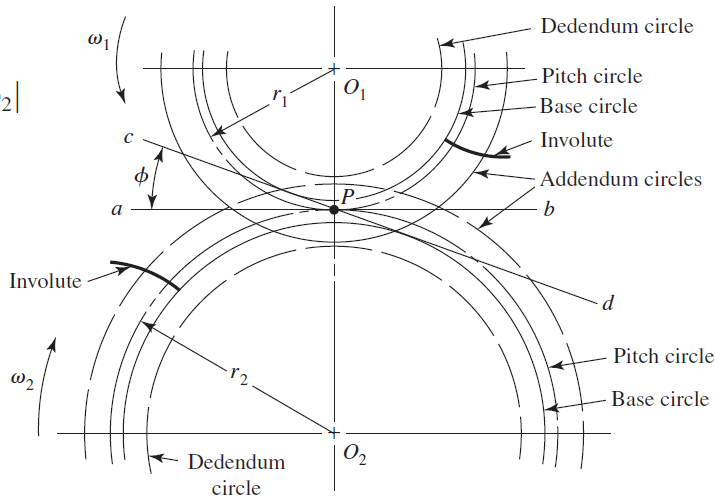
Pitch circles roll without slipping and are the basis of the various dimensions found in gearing.

$$r_b = r \cos \phi$$

$$V = |r_1 \omega_1| = |r_2 \omega_2|$$



$$\left| \frac{\omega_1}{\omega_2} \right| = \frac{r_2}{r_1}$$

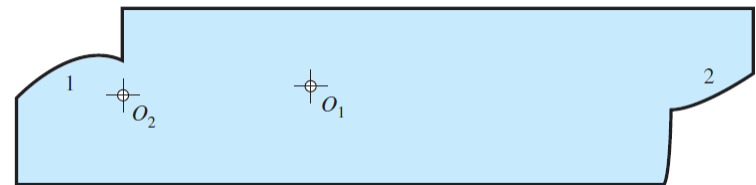


1. Locate gear centers O_1 and O_2 ;
2. Construct **pitch circles** of radii r_1 and r_2 ;
3. Identify tangent at P , the **pitch point**;
4. Draw line ab , the common tangent through P ;
5. Draw line cd through P at angle ϕ to ab ;

Pressure Line
/Generating Line/Line of Action

Pressure Angle ($20^\circ/25^\circ$)

6. On each gear draw a **base circle** tangent to the pressure line, determined by ϕ ;
7. Now generate an **involute** on each base circle as previously described;
8. Use $1/P$ for the addendum distance and $1.25/P$ for the dedendum distance to draw the **addendum** and **dedendum** circles on the pinion and on the gear.
9. Determine circular pitch using $p = \pi/P$;
10. Determine tooth thickness using $t = p/2$;
11. Prepare a template to draw the profile;
12. Finish the drawing by adding the fillets between the **clearance circle** and the dedendum circle.

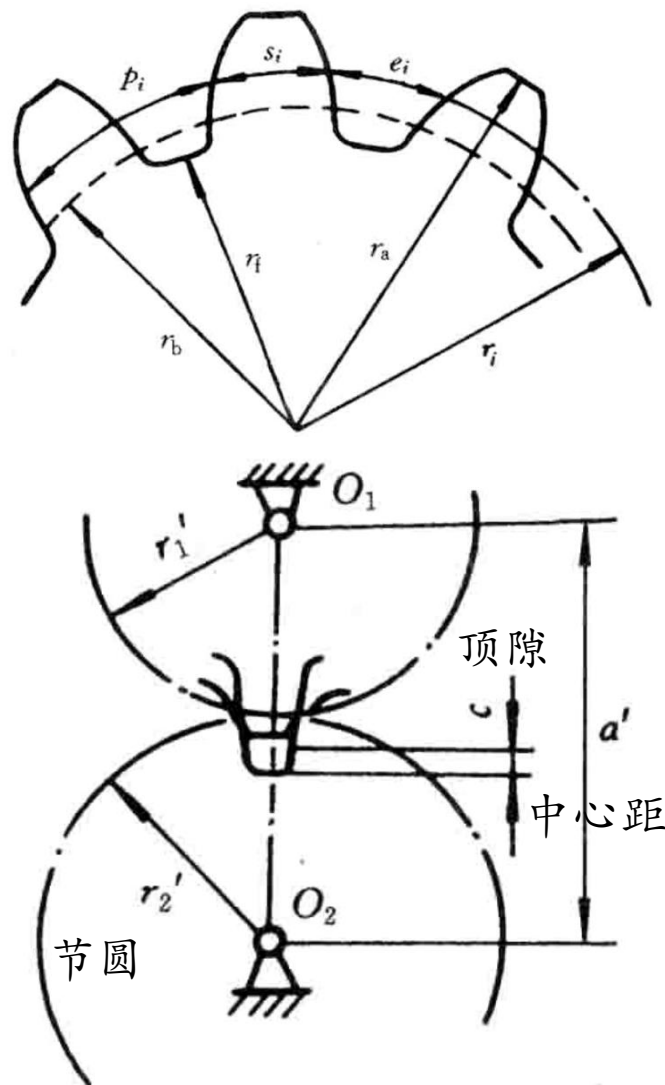


渐开线直齿 圆柱齿轮机构

齿轮机构及其设计

外啮合直齿圆柱齿轮机构的基本参数和尺寸

- 1) 齿数 z
 - 在齿轮的整个圆周上轮齿的总数称为齿数，常用 z 表示
- 2) 齿槽宽 e_i
 - 齿轮上两相邻轮齿之间的空间称为齿间或齿槽
 - 一齿槽两侧齿廓间在任意圆周上的弧长，称为该圆上的齿槽宽（简称齿宽），用 e_i 表示
- 3) 齿厚 s_i
 - 在任意半径的圆周上，一个轮齿两侧齿廓间的弧长，称为该圆上的齿厚，用 s_i 表示
- 4) 齿距 $p_i = s_i + e_i$
 - 相邻两齿同侧两齿廓间在某一圆上的弧长，称为该圆上的齿距，用 p_i 表示
 - 在同一圆周上，齿距等于齿厚和齿槽宽之和
- 5) 顶隙（径向间隙） c
 - 顶隙是指一对相啮合齿轮中，一齿轮的齿根圆与另一齿轮齿顶圆之间在连心线上度量的距离，用 c 来表示
 - 留有一定的顶隙是为了避免一齿轮的齿顶与另一齿轮的齿槽相抵触发生干涉，同时也便于储存润滑油



模数

- 设一齿数为 z 的齿轮，其任一圆上的直径为 d_i ，该圆上的齿距为 p_i ，则有 $d_i = 2r_i = \frac{p_i z}{\pi}$
 - z 为正整数， π 为无理数
 - 若选 p_i （有理数）作为基本入参数，则直径 d_i 为无理数，这不符合工程设计和制造的要求
- 为了便于设计、制造和检测，**人为地**把 $\frac{p_i}{\pi}$ 规定为一简单的有理数，并把这个比值称为模数，用 m_i 来表示，即 $m_i = \frac{p_i}{\pi}$

分度圆

- 为了使基本参数具有单一化的确定性，在齿轮计算中必须规定一个圆作为尺寸计算的基准圆，这个圆就称为分度圆，其直径和半径分别用 d 和 r 表示
 - 虽以 m_i 为基本参数可使 d_i 不是无理数，但一个齿轮在不同直径的圆周上，其模数大小是不同的
- 我国国家标准规定，分度圆上的模数和压力角为标准值
 - 因为一种模数和压力角的齿轮，需要用一把专用的刀具进行加工，为了协调齿轮设计和刀具生产之间的矛盾，各国均制定了相应的标准
 - 我国国家标准规定的压力角标准值为 20° ，模数的标准系列如下表所示

表 4-2 标准模数系列

单位: mm

第一系列	1	1.25	1.5	2	2.5	3	4	5	6	8	10	12	16	20	25	32	40	50
第二系列	1.75	2.25	2.75	(3.25)	3.5	(3.75)	4.5	5.5	(4.5)	7	9	(11)	14	18	22	28	36	45

注：① 优先采用第一系列，括号内的数值尽量不用；② 在采用英制单位的国家，以径节来计算齿轮基本尺寸，径节 p 是齿数 z 与分度圆直径 d 之比，即 $d = z/p$ 。径节与模数之间具有如下关系： $m = 25.4/p$ 。

分度圆

- 分度圆是齿轮上一个人为约定的用于计算的基准圆
 - 通常，分度圆就是齿轮上具有标准模数和标准压力角的圆
 - 任何一个齿轮都有且仅有一个分度圆，其值为 $d = mz$
- 以后，凡未说明是哪个圆上的模数、齿距、齿厚、齿槽宽和压力角，都是指分度圆上的，并分别用 m 、 p 、 s 、 e 和 a 表示
 - 若是其他圆上的参数则需指明
- 齿数、模数和压力角是齿轮尺寸计算中的三个基本参数
 - 模数大，则齿轮的轮齿大；模数一定时，齿数多，则齿轮的轮齿大

表 4-2 标准模数系列

单位: mm

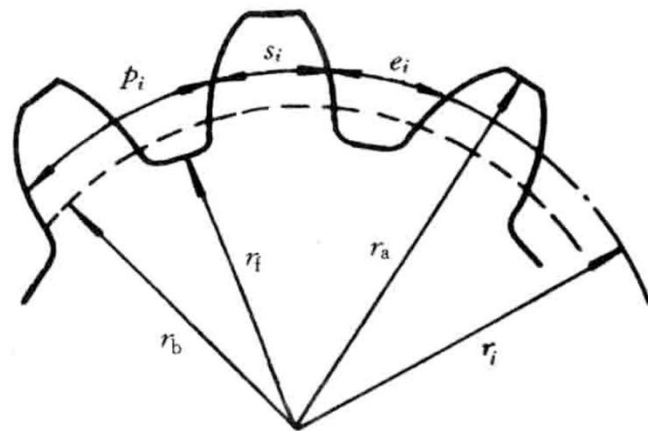
第一系列	1	1.25	1.5	2	2.5	3	4	5	6	8	10	12	16	20	25	32	40
	50																
第二系列	1.75	2.25	2.75	(3.25)	3.5(3.75)	4.5	5.5(4.5)	7	9								
	(11)	14	18	22	28	36	45										

注：① 优先采用第一系列，括号内的数值尽量不用；② 在采用英制单位的国家，以径节来计算齿轮基本尺寸，径节 p 是齿数 z 与分度圆直径 d 之比，即 $d = z/p$ 。径节与模数之间具有如下关系： $m = 25.4/p$ 。

基圆、齿顶圆和齿根圆

• 1) 基圆

- 基圆的大小是决定渐开线形状的唯一条件
 - 基圆直径 $d_b = d \cos \alpha = mz \cos \alpha$
 - 基圆上的齿距（或称基节） $p_b = \frac{\pi d_b}{z} = \pi m \cos \alpha$



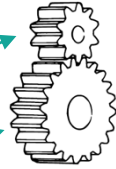
• 2) 齿顶圆和齿根圆

- 齿顶圆和齿根圆一般分别处于分度圆的两侧
 - 分度圆与齿顶圆之间的径向距离称为齿顶高 h_a : $d_a = d + 2h_a$
 - 分度圆与齿根圆之间的径向距离称为齿根高 h_f : $d_f = d - 2h_f$
 - 齿顶圆与齿根圆之间的径向距离称为齿高 h : $h = h_a + h_f$

Nomenclature

The pitch circles of a pair of mating gears are tangent to each other.

- A **pinion** is the smaller gear
- The larger is often called the **gear**



The distance, measured on the pitch circle, from a point on one tooth to a corresponding point on an adjacent tooth

- Tooth thickness + width of space

$$P = \frac{N}{d} \quad p = \frac{\pi d}{N} = \pi m$$

$$m = \frac{d}{N} \quad pP = \pi$$

P = diametral pitch, teeth per inch

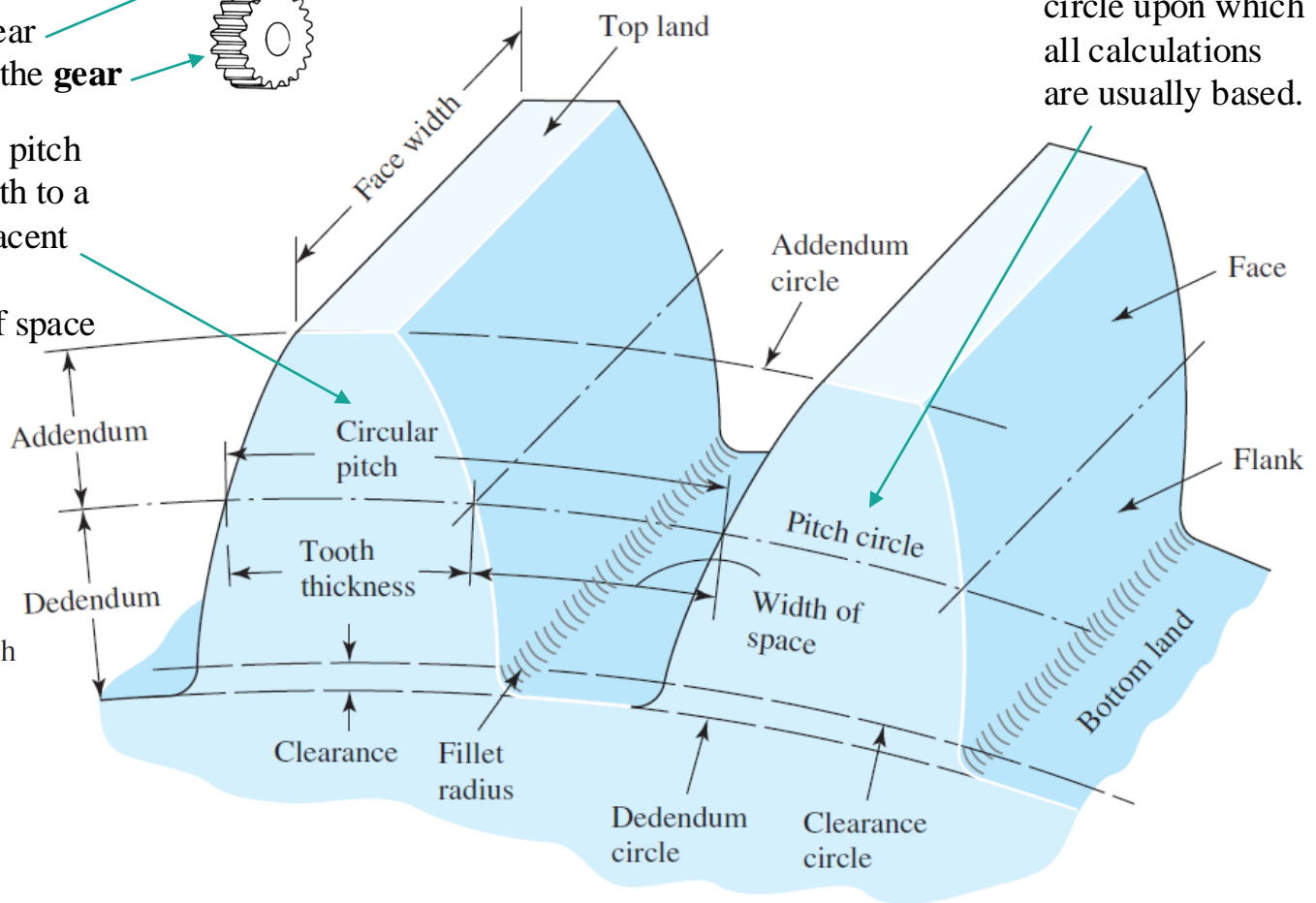
N = number of teeth

d = pitch diameter, in or mm

m = module, mm

p = circular pitch, in or mm

A theoretical circle upon which all calculations are usually based.



Nomenclature

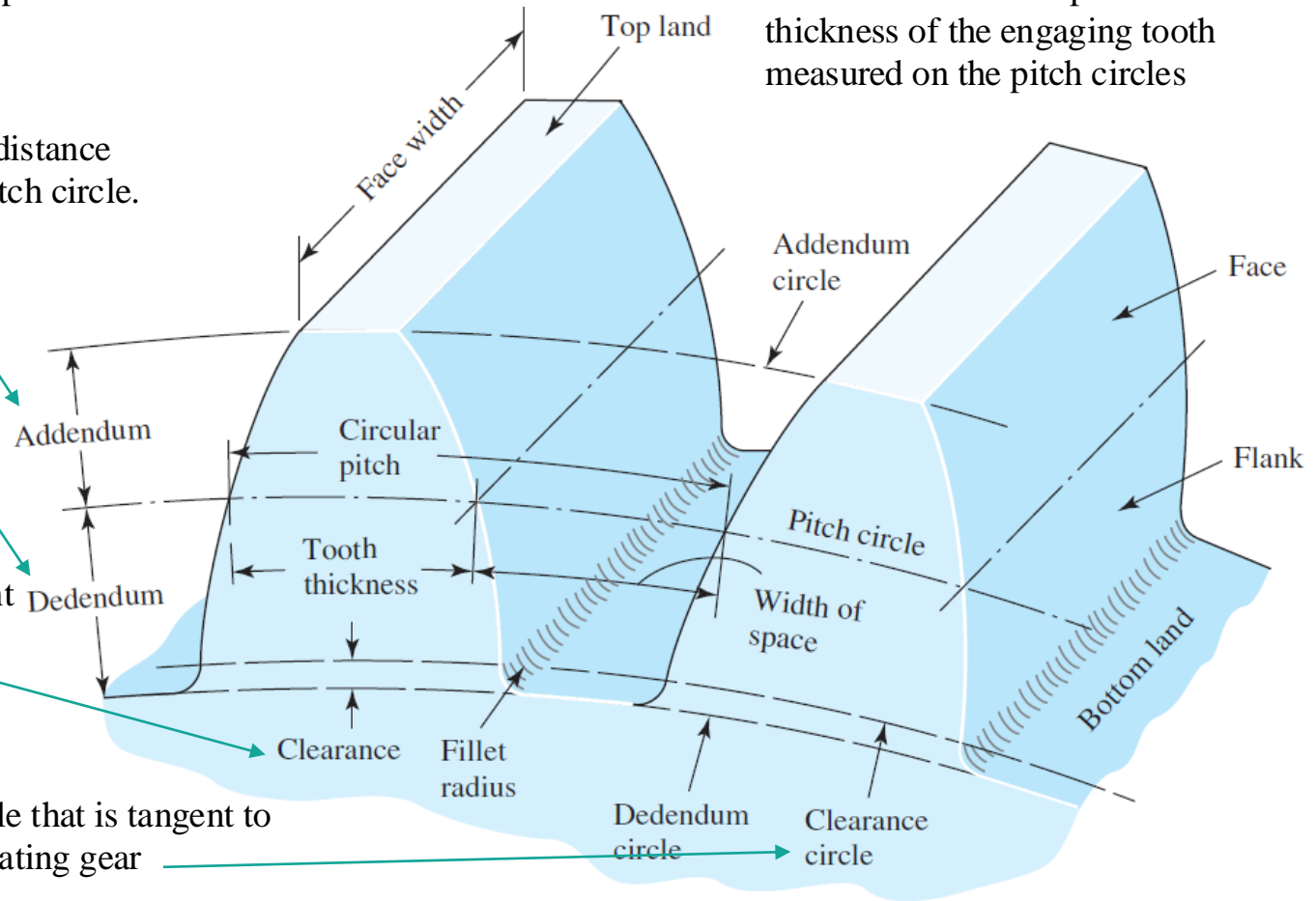
The *addendum* a is the radial distance between the *top land* and the pitch circle.

The *backlash* is the amount by which the width of a tooth space exceeds the thickness of the engaging tooth measured on the pitch circles

The *dedendum* b is the radial distance from the *bottom land* to the pitch circle.

The *clearance* c is the amount by which the dedendum in a given gear exceeds the addendum of its mating gear.

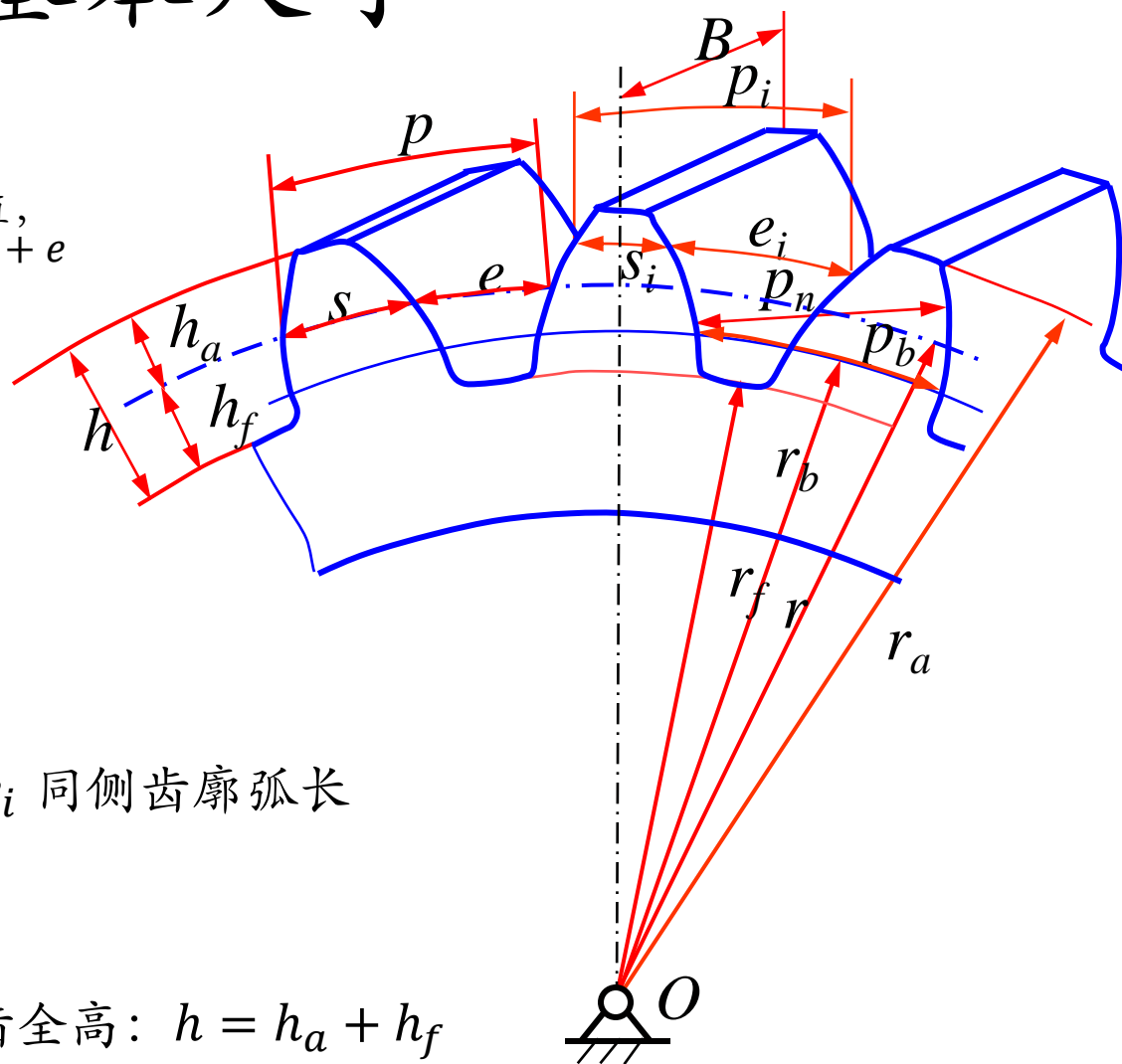
The *clearance circle* is a circle that is tangent to the addendum circle of the mating gear



基本尺寸

• 分度圆 (d 、 r)

- 人为规定的计算基准圆，此圆上的模数和压力角为标准值，表示符号： d 、 r 、 s 、 e ， $p = s + e$

• 齿顶圆 (d_a 、 r_a)• 基圆 (d_b 、 r_b)• 齿根圆 (d_f 、 r_f)• 齿厚： s_i 任意圆上的弧长• 齿槽宽： e_i 弧长• 齿距 (周节)： $p_i = s_i + e_i$ 同侧齿廓弧长• 法向齿距 (法节)： $p_n = p_b$ • 齿顶高： h_a | 齿根高： h_f | 齿全高： $h = h_a + h_f$ • 齿宽： B 

外啮合标准直齿圆柱齿轮机构的几何尺寸计算

• 1. 标准齿轮

- 模数 m 和压力角 α 为**标准值**：查表
- 分度圆上的齿厚 s 与齿槽宽 e 相等： $s = e = \frac{1}{2}p = \frac{1}{2}\pi m$
- 齿顶高 h_a 与模数的比值等于**标准值**： $\frac{h_a}{m} = h_a^* = 1$ ， h_a^* 齿顶高系数
- 齿根高 h_f 与模数的比值均等于**标准值**： $\frac{h_f}{m} = h_a^* + c^* = 1 + 0.25$ ， c^* 顶隙系数
- **根据我国基本齿廓标准 (GB/T1356-2001)**
 - $s = e = \frac{1}{2}p = \frac{1}{2}\pi m$ ， $c = c^*m$ ， $h_a = h_a^* m$ ， $h_f = (h_a^* + c^*)m$

外啮合标准直齿圆柱齿轮机构的几何尺寸计算

• 2. 标准齿轮传动的中心距

- 中心距是齿轮传动的基本尺寸，齿轮箱体上轴承孔的尺寸就是由中心距决定的
 - 为了使一对渐开线标准齿轮传动平稳，在确定其中心距时，应保证相啮合的两轮齿的齿侧无间隙存在
- 一对齿轮啮合传动时，两齿轮的中心距总等于作相对滚动的两节圆的半径之和
 - 当要求相啮合的两轮齿齿侧无间隙存在时，一齿轮轮齿的节圆齿厚必须等于另一齿轮轮齿的节圆齿槽宽，故一般称 $s'_1 = e'_2$, $s'_2 = e'_1$ 为 **齿轮机构的无侧隙传动条件**
- 由于一对模数相等、无侧隙啮合的标准齿轮分度圆上的齿厚和齿槽宽相等，即 $s_1 = e_1 = s_2 = e_2 = \pi m/2$
 - 因而，当两轮的分度圆相对滚动时，其齿侧间隙为零，此时，分度圆与节圆重合
 - 因而，一对无侧隙啮合的标准齿轮，其中心距（简称为标准中心距，用 a 表示）的计算式为 $a = r'_1 + r'_2 = r_1 + r_2 = (z_1 + z_2) m/2$

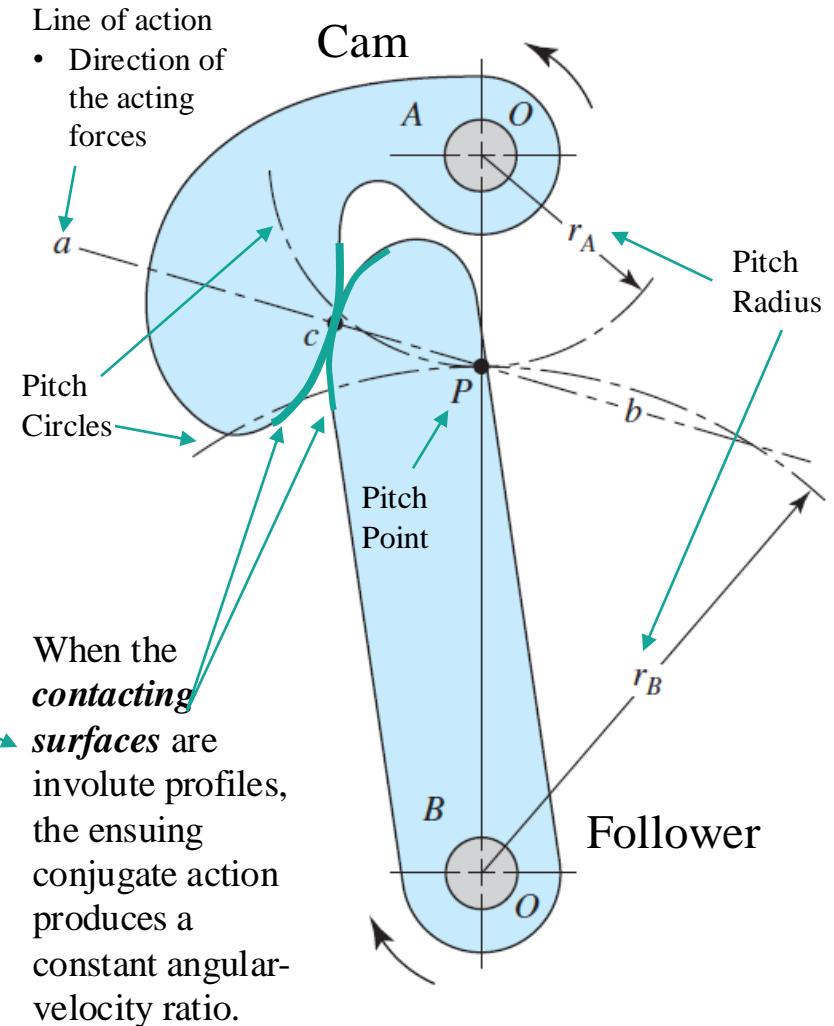
外啮合标准直齿圆柱齿轮机构的几何尺寸计算

表 4-3 外啮合标准直齿圆柱齿轮几何尺寸的计算公式

名 称	符 号	计 算 公 式
模数	m	根据齿轮强度等要求选择的标准值
压力角	α	$\alpha = 20^\circ$
分度圆直径	d	$d_1 = mz_1, \quad d_2 = mz_2$
齿顶高	h_a	$h_a = h_a^* m$
齿根高	h_f	$h_f = (h_a^* + c^*) m$
齿高	h	$h = h_a + h_f$
顶隙	c	$c = c^* m$
齿顶圆直径	d_a	$d_{a1} = d_1 + 2h_a, \quad d_{a2} = d_2 + 2h_a$
齿根圆直径	d_f	$d_{f1} = d_1 - 2h_f, \quad d_{f2} = d_2 - 2h_f$
基圆直径	d_b	$d_{b1} = d_1 \cos\alpha, \quad d_{b2} = d_2 \cos\alpha$
齿距	p	$p = \pi m$
齿厚	s	$s = \frac{p}{2}$
齿槽距	e	$e = \frac{p}{2}$
标准中心距	a	$a = \frac{1}{2}(z_1 + z_2)m$

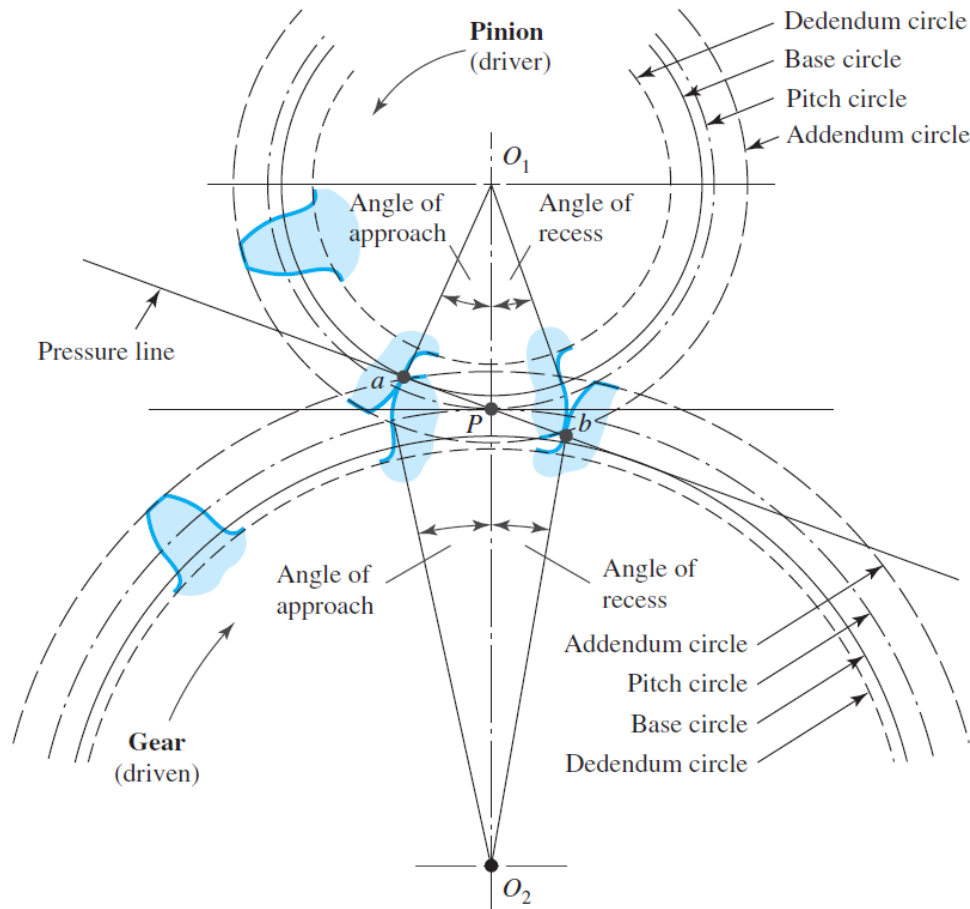
Conjugate Action

- Mating Gears vs. Cams
- Conjugate Action
 - When the tooth profiles, or cams, are designed so as to produce *a constant angular-velocity ratio* during meshing.
- Involute Profile
 - In universal use for gear teeth and with few exceptions.
- Features
 - Conjugate action is independent of changes in center distance
 - Can be manufactured at low cost since the tooth profile is relatively simple
 - A typical tooth profile used almost exclusively for gears

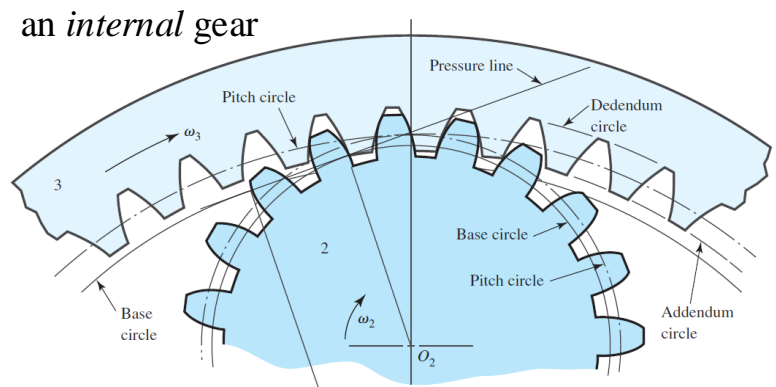
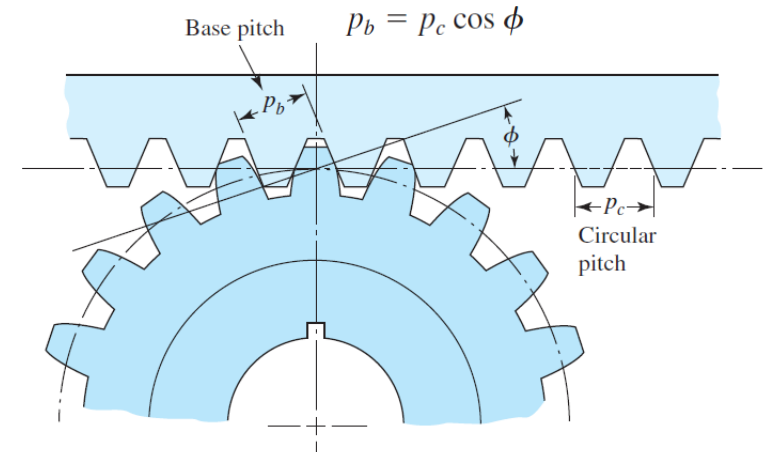


Tooth Action

The motion of one tooth relative to the mating tooth is *primarily* a rolling motion;
In fact, when contact occurs at the *pitch point*, the motion is *pure rolling*.



We may imagine a *rack* as a spur gear having an infinitely large pitch diameter.



Contact Ratio

- Contact Ratio

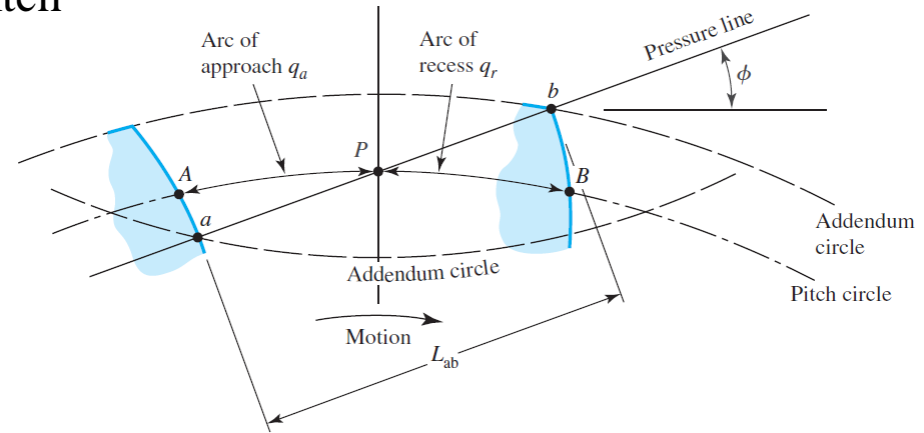
$$m_c = \frac{q_t}{p}$$

Arc of Action $q_t = q_a + q_r$

Circular pitch

- Gears should not generally be designed having contact ratios less than about 1.20,
 - Inaccuracies in mounting might reduce the contact ratio even more,
 - Increasing the possibility of impact between the teeth
 - As well as an increase in the noise level.
- An easier way to obtain the contact ratio is to measure the line of action ab instead of the arc distance AB .

$$m_c = \frac{L_{ab}}{p \cos \phi}$$



Tooth contact *begins* and *ends* at the intersections of the two addendum circles with the pressure line, i.e. A and B .

渐开线直齿圆柱齿轮机构的啮合传动

1. 能实现恒定的瞬时角传动比传动

- 设两渐开线齿廓 E_1 、 E_2 在任意点 K_1 相啮合
 - 过点 K_1 作这对齿廓的公法线 N_1N_2 与两轮的连心线交于点 P
 - 由渐开线的性质可知，此公切线 N_1N_2 必与两基圆相切，即 N_1N_2 必为两基圆的内公切线
 - 因在传动过程中，两基圆为定圆，而两定圆在同一方向的内公切线只有一条，故两齿廓无论在何处啮合，过啮合点所作的两齿廓的公法线与两基圆的内公切线重合
 - 由此可知，两齿廓在所有啮合点上的公法线都通过连心线上的定点 P

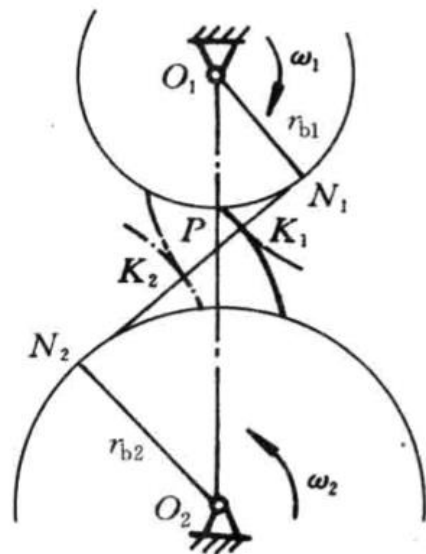


图 4-5 能实现恒定的瞬时角传动比传动

- 因而有 $i_{12} = \frac{\omega_1}{\omega_2} = \frac{O_2P}{O_1P} = \text{const}$

- 这就证明了渐开线齿廓能保证实现恒定的瞬时传动比传动

- $i_{12} = \frac{\omega_1}{\omega_2} = \frac{O_2P}{O_1P} = \frac{r_2'}{r_1'} = \frac{r_{b2}}{r_{b1}} = \frac{r_2 \cos \alpha}{r_1 \cos \alpha} = \frac{z_2}{z_1}$

- r_1' 、 r_2' 分别为两轮的节圆半径； r_{b1} 、 r_{b2} 分别为两轮的基圆半径

渐开线直齿圆柱齿轮机构的啮合传动

• 2. 中心距具有可分性

- 两轮中心距的变化不影响传动的瞬时角速比，这一特性称为中心距的可分性
- 两轮的瞬时角速比不仅与两轮的节圆半径成反比，而且与基圆半径成反比

$$\bullet i_{12} = \frac{\omega_1}{\omega_2} = \frac{\overline{O_2P}}{\overline{O_1P}} = \frac{r'_2}{r'_1} = \frac{r_{b2}}{r_{b1}} = \frac{r_2 \cos \alpha}{r_1 \cos \alpha} = \frac{z_2}{z_1}$$

- 即使两轮的实际中心距与设计中心距有点偏差，也不会改变其瞬时角速比

$$\bullet i'_{12} = \frac{\omega_1}{\omega'_2} = \frac{\overline{O'_2P'}}{\overline{O_1P}} = \frac{r_{b2}}{r_{b1}}$$

- 两轮中心距的变化只改变两轮的节圆半径，齿轮制成后，其基圆就已确定，不因中心距的变化而有所改变

- 渐开线齿廓啮合的一大优点，有很大的实用价值
 - 实际工作中，由于制造和安装误差，以及轴承磨损等原因，齿轮的实际中心距与设计中心距往往不相等，但由于渐开线齿廓啮合具有中心距的可分性，故仍可保持定传动比传动

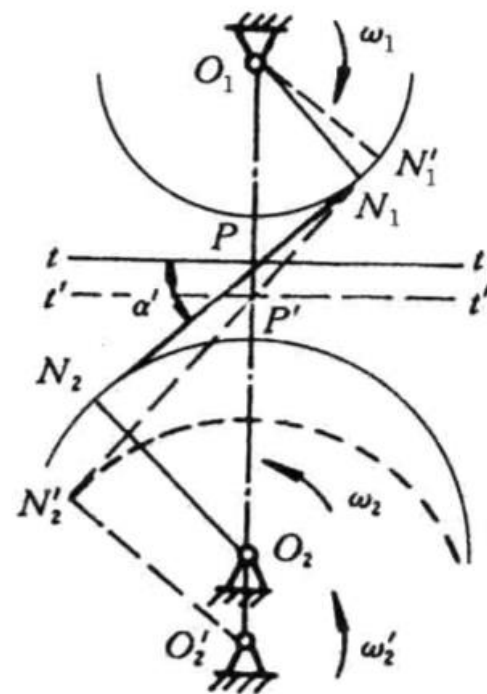
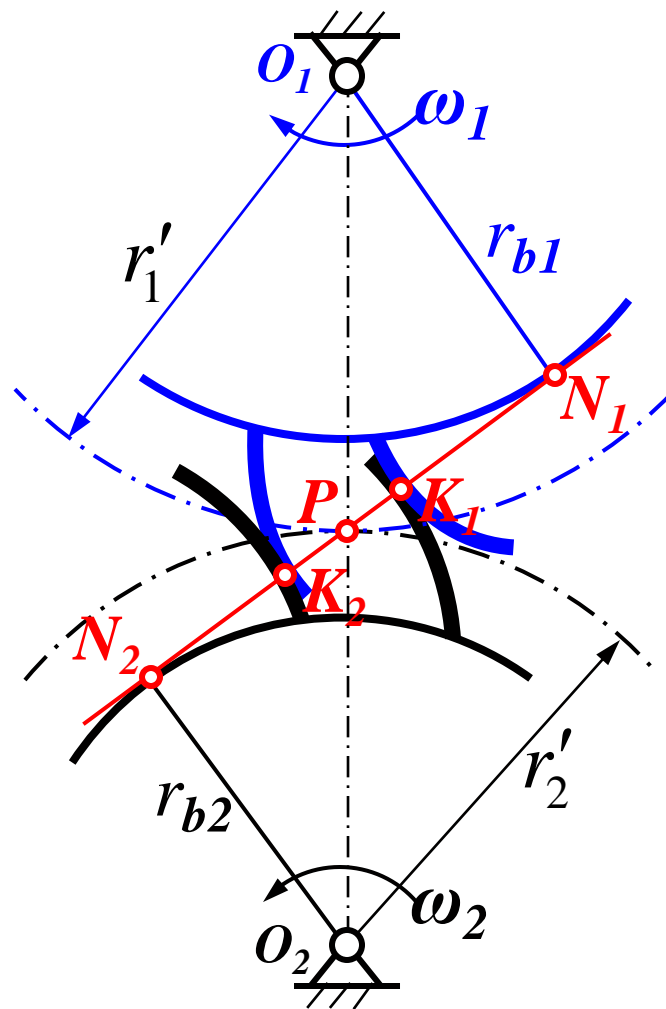


图 4-6 中心距的可分性

渐开线直齿圆柱齿轮机构的啮合传动

• 3. 啮合线是两基圆上的一条内公切线

- 两渐开线齿廓在任何位置啮合时，过啮合点所作的齿廓公法线均为直线 N_1N_2
- 因此，在渐开线齿廓啮合过程中，其每个瞬时的接触点都在直线 N_1N_2 上
- 啮合线：两齿廓啮合点在与机架相固连的坐标系中的轨迹
- 啮合线、齿廓接触点的公法线、正压力方向线都是两基圆的一条内公切线



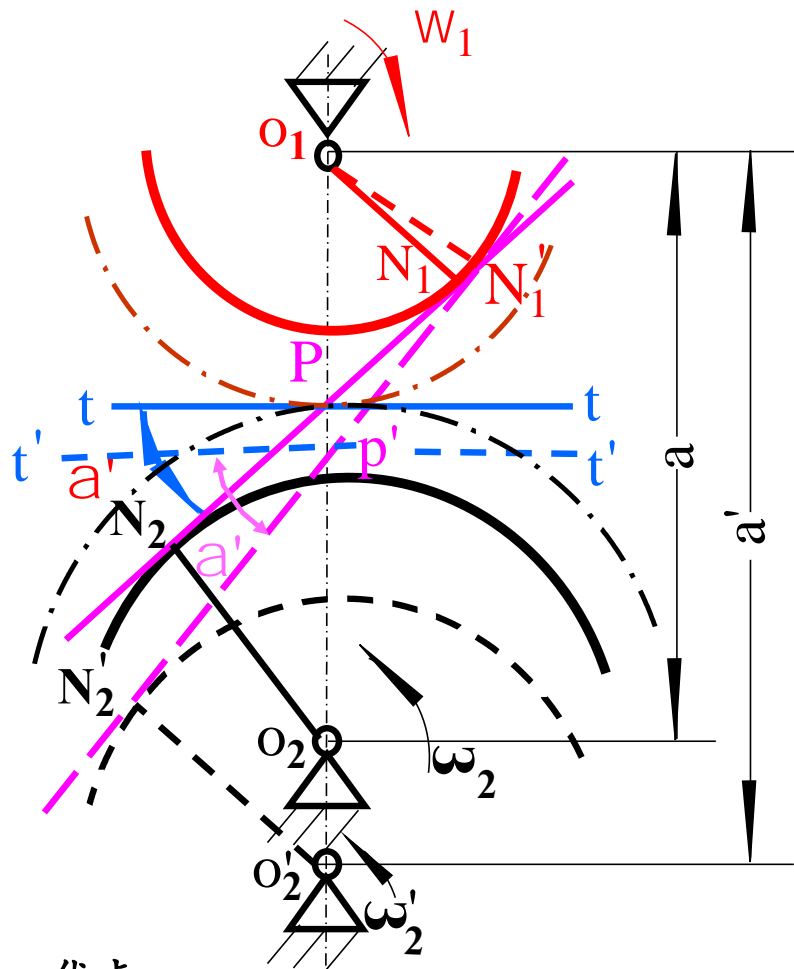
渐开线直齿圆柱齿轮机构的啮合传动

4. 啮合角是随中心距而定的常数

- 两齿廓在啮合过程中，过节点所作的两节圆的内公切线 $t-t$ 与两齿廓接触点的公法线所夹的锐角，称为啮合角，一般用 α' 表示

$$\cos \alpha' = \frac{r_{b1}}{r_1'} = \frac{r_{b2}}{r_2'}$$

- 渐开线齿廓的啮合角等于该对齿廓在节点接触时的压力角，与齿轮的基圆和节圆半径有关
- 一对安装好的渐开线圆柱齿轮，其节圆和基圆的位置确定不变，因而，啮合角在渐开线齿廓的啮合过程中是恒定不变的
 - 但因中心距加大时，节圆半径随之加大，所以，啮合角随中心距的变化而改变，标注中心距时，其啮合角数值上等于分度圆压力角



啮合角在啮合过程中恒定不变，是渐开线齿轮传动的又一优点

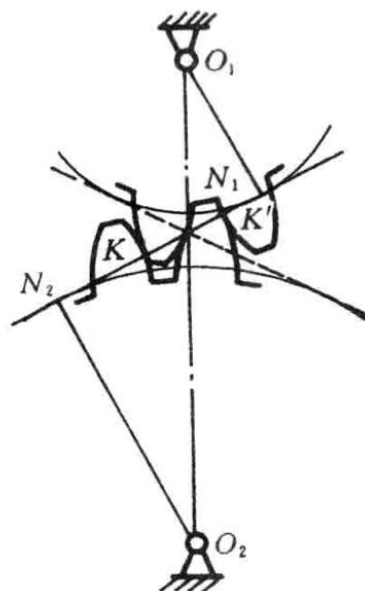
- 齿轮传动时，两齿廓间的正压力沿齿廓接触点的公法线方向作用，因啮合过程中啮合角为一不变的常数，故两齿廓间的正压力方向在啮合过程中始终保持不变
- 当主动轮上的驱动力矩 T_1 为常数时，作用在从动齿轮齿廓上的正压力的方向和大小均不变，这对支撑齿轮的轴承的受力情况十分有利

渐开线直齿圆柱齿轮机构的啮合传动

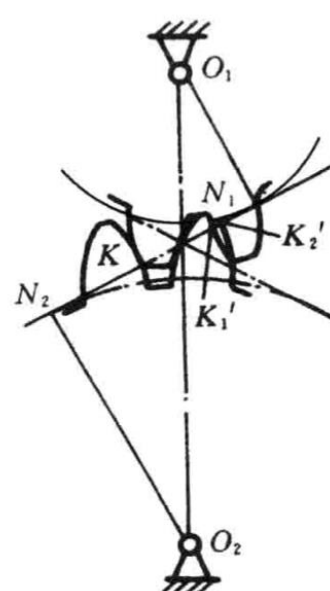
5. 正确啮合条件

- 一对渐开线齿廓能够保证定传动比传动，但并不是任意两个渐开线齿轮搭配起来都可以正确地传动

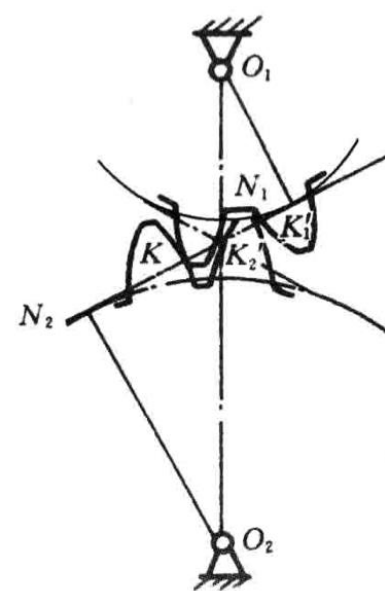
- 齿轮传动是靠齿轮上的轮齿一对对地依次啮合来实现的，每一对齿只能在有限的区间内啮合，随后便要分离，由后一对齿接替



(a)



(b)



(c)

- 一对渐开线齿轮传动时，其啮合点都应在啮合线上

当前一对齿在啮合线上的点K接触时，如果后一对齿也处于啮合线上，则它们必须在啮合线上的一点K'接触

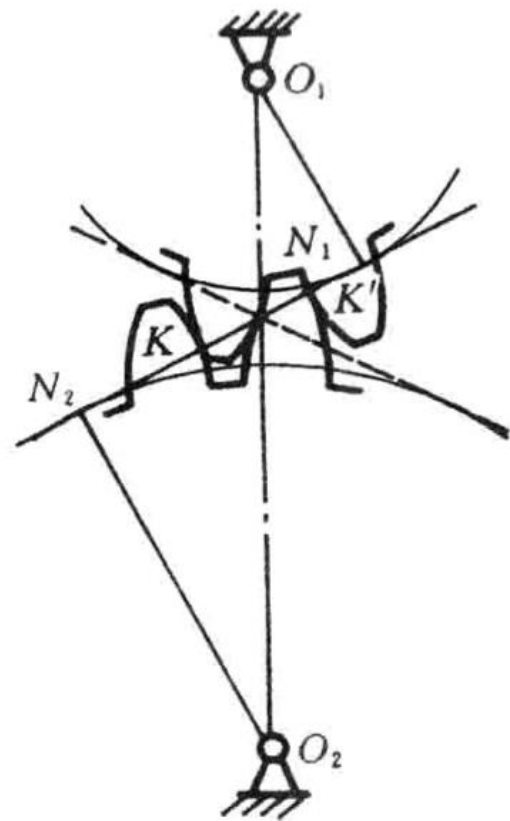
否则，将会出现齿廓重叠（由于两轮齿是刚体，不能相互嵌入，势必使这一对齿轮无法安装，不能进行正常啮合）

存在齿侧间隙（使得当前一对齿轮在啮合线上接触终止时，下一对轮齿不能及时地进行接触，不能保证实现定传动比传动），这都是不允许的

渐开线直齿圆柱齿轮机构的啮合传动

• 5. 正确啮合条件

- 为了满足这一要求，必须使轮 1 和轮 2 上相邻两齿同侧齿廓的法向距离（称为齿轮的法节）相等
 - $K'K$ 既是齿轮 1 的法节，又是齿轮 2 的法节
 - 因此，只有两齿轮的法节相等，它们才能正确啮合
- 根据渐开线的性质，齿轮的法节与其端面基圆齿距（基节）在数值上相等，于是得 $p_{b1} = p_{b2}$
 - 渐开线齿轮传动的正确啮合条件
 - $$\left. \begin{aligned} p_{b1} &= \pi m_1 \cos \alpha_1 \\ p_{b2} &= \pi m_2 \cos \alpha_2 \end{aligned} \right\} \Rightarrow m_1 \cos \alpha_1 = m_2 \cos \alpha_2$$
 - 由于齿轮的模数和压力角都已标准化了，应使
 - $$\left. \begin{aligned} m_1 &= m_2 = m \\ \alpha_1 &= \alpha_2 = \alpha \end{aligned} \right\}$$
- 两轮的模数和压力角必须分别相等是一对渐开线齿轮的正确啮合条件



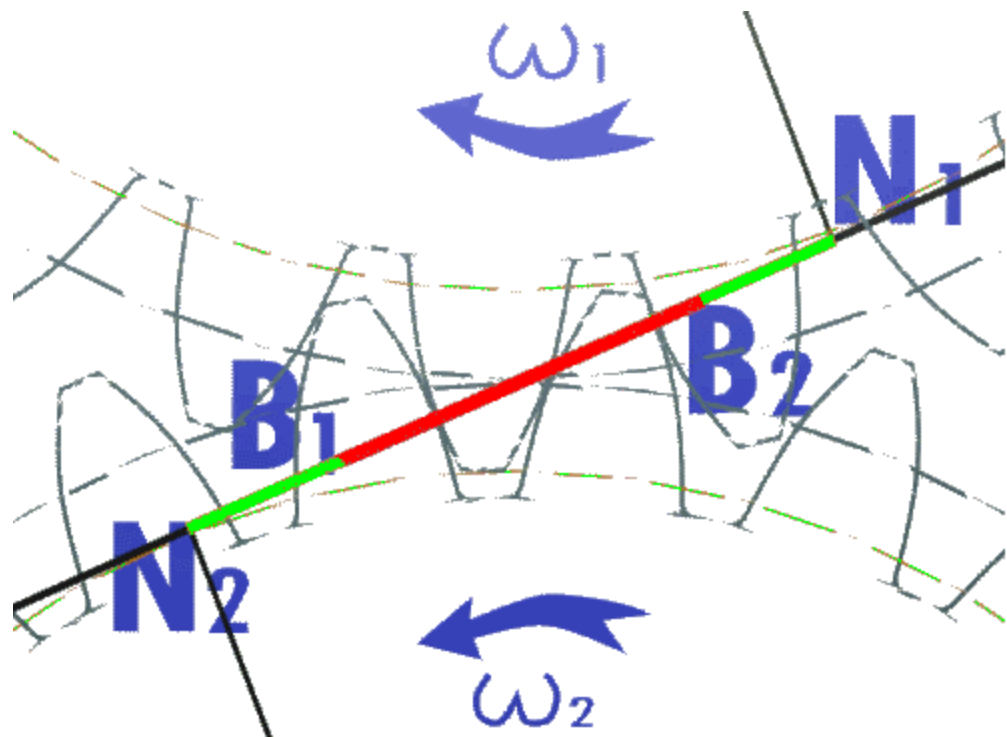
问：美国齿轮与中国齿轮能正确啮合吗？

答：两国标准不一样

渐开线直齿圆柱齿轮机构的啮合传动

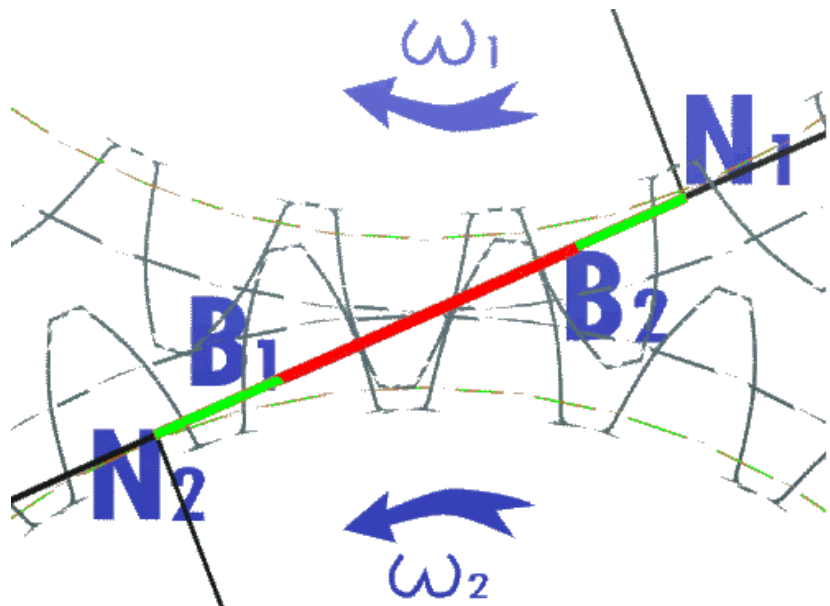
6. 连续传动条件：1) 一对渐开线直齿圆柱齿轮机构的啮合传动过程

- 图示为一对渐开线齿轮的啮合传动，设齿轮 1 为主动轮，齿轮 2 为从动轮，转向如图所示
- 在正常情况下，当两轮齿开始啮合时，必为主动轮的根部齿廓与从动轮的齿顶相接触
 - 由于齿廓接触点必在啮合线上，所以一对轮齿在啮合线上的起点，就是从动轮 2 的齿顶圆与啮合线 N_1N_2 的交点 B_2
 - 随着啮合传动的进行，接触点便由点 B_2 沿着啮合线向 N_2 的方向移动，直到主动轮 1 的齿顶与从动轮 2 的齿根部齿廓相接触（如图中虚线所示位置）时，两齿廓即将脱离在啮合线 N_1N_2 上的接触
 - 所以，一对轮齿在啮合线上啮合的终止点就是主动轮 1 的齿顶圆与啮合线 N_1N_2 的交点 B_1

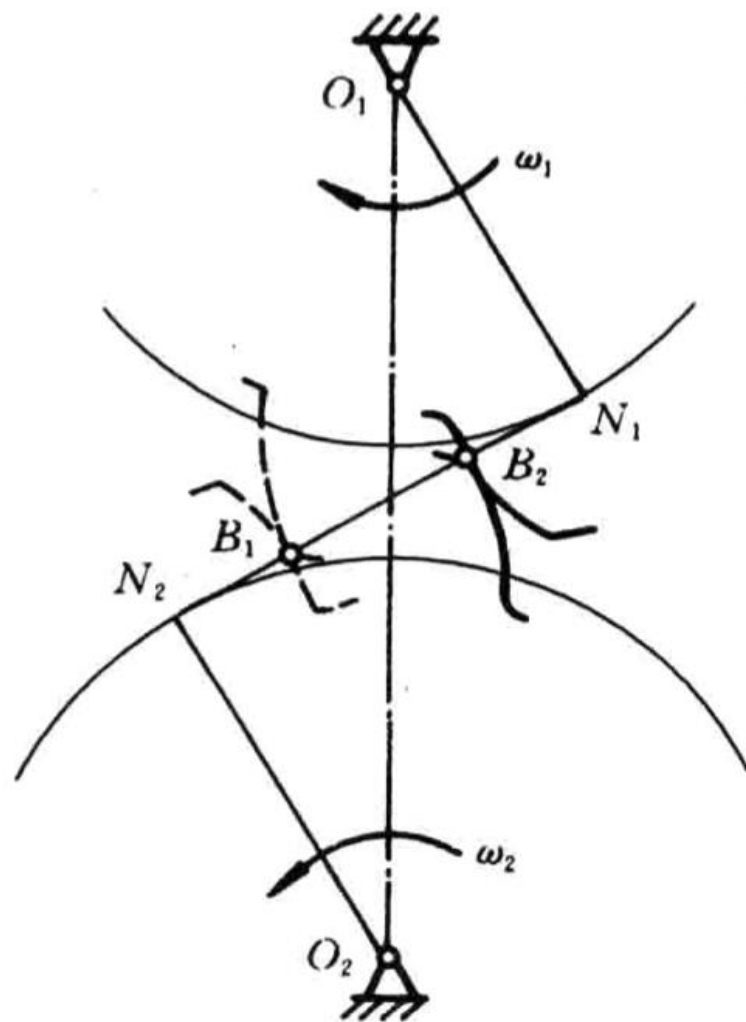


渐开线直齿圆柱齿轮机构的啮合传动

- 6. 连续传动条件：1) 一对渐开线直齿圆柱齿轮机构的啮合传动过程



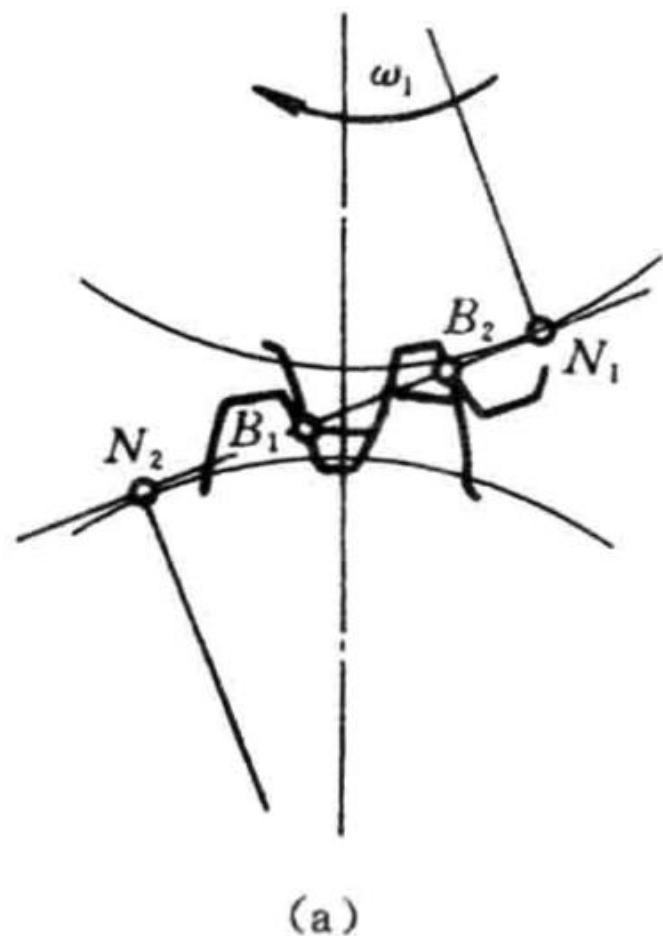
- 实际啮合线：线段 B_1B_2 是一对轮齿啮合点在与机架固连的坐标系上的实际轨迹
- 若将两轮的齿顶圆加大，则点 B_1 和 B_2 将分别趋近于啮合线与两基圆的切点 N_1 和 N_2 ，因基圆内没有渐开线，所以，两轮齿顶圆与啮合线的交点不得超过点 N_1 和 N_2
- 理论啮合线：线段 N_1N_2 是理论上可能的最长啮合线段



渐开线直齿圆柱齿轮机构的啮合传动

• 6. 连续传动条件：2) 重合度

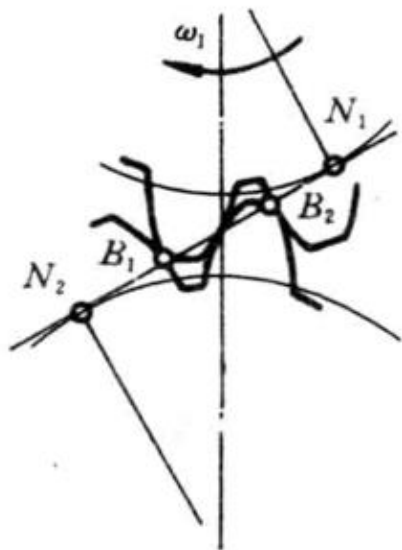
- 为了避免冲击、振动以及减小噪声，要求直齿圆柱齿轮能保持连续定角速比传动
- 如下现象是齿轮传动所不允许的
 - 当主动轮1的齿顶与从动轮2的齿根部在啮合线上的点 B_1 接触时，其后面的一对轮齿还没有接触
 - 于是当主动轮1再继续等速转动时，前一对轮齿将不再在啮合线 N_1N_2 上接触，而是主动轮1的齿顶尖角在从动轮2的齿廓上滑过去，推动从动轮2减速转动，直到后一对轮齿在啮合线上的点 B_2 （从动轮齿顶圆与 N_1N_2 的交点）接触时，才又作定角速比啮合，此时前一对轮齿便脱离接触
- 产生这种现象的原因是实际啮合线 $\overline{B_1B_2}$ 的长度小于齿轮的法节（即 $\overline{B_1B_2} < p_n$ ），致使前一对轮齿到达啮合线上脱离接触的位置时，后一对轮齿还未能在啮合线上进入接触的缘故



渐开线直齿圆柱齿轮机构的啮合传动

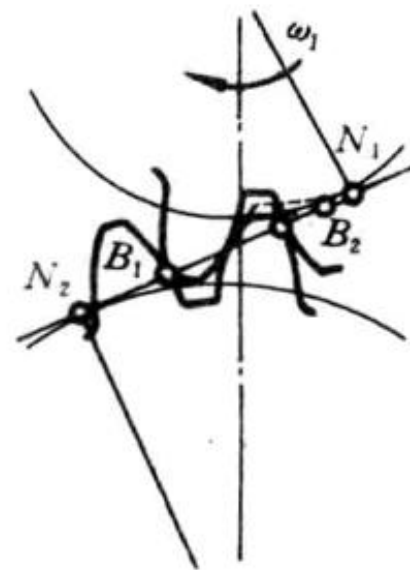
6. 连续传动条件：2) 重合度 ε_a

- 图示为一对恰好能保证连续定角速比传动的渐开线齿轮的工作情况，当前一对轮齿在点 B_1 即将脱离接触时，由于 $\overline{B_1B_2} = p_n$ ，故后一对轮齿刚好在点 B_2 开始接触



保证连续定角速比传动的条件为 $\overline{B_1B_2} > p_n$ ，或
 写为 $\varepsilon_a = \frac{\overline{B_1B_2}}{p_n} \geq 1$

- 图示为一对渐开线齿轮为 $\overline{B_1B_2} > p_n$ 的啮合情况，此时前一对轮齿在点 B_1 即将脱离接触时，后一对轮齿早已处于接触状态，因此就能保证连续定角速比传动



- 因此当主动轮 1 再继续等速转动时，前一对轮齿虽已脱离接触，但后一对轮齿已经在啮合线上正常地啮合，因而可保证从动轮 2 按定角速比等速转动

- 一般称比值 ε_a 为齿轮传动的重合度，重合度的大小是表明同时参与啮合的轮齿对数的多少
 - 例如当 $\varepsilon_a = 1$ 时，表示在齿轮传动过程中，除了在点 B_1 和 B_2 接触瞬间

渐开线直齿圆柱齿轮机构的啮合传动

6. 连续传动条件：2) 重合度

- 当 $\varepsilon_a = 1.3$ 时，直线 $\overline{B_1B_2}$ 表示实际啮合线长度，即由点 B_1 和 B_2 分别量取等于法节 p_n 的线段 $\overline{B_1K}$ 和 $\overline{B_2K'}$
 - 假如第一对轮齿在位置 K' 接触，由于 $\overline{B_2K'} = p_n$ ，所以第二对轮齿在位置 B_2 刚刚进入啮合，在此以后，为两对轮齿同时啮合
 - 当第一对轮齿到达位置 B_1 时，由于 $\overline{B_1K} = p_n$ ，所以第二对轮齿到达位置 K
 - 当第一对轮齿自点 B_1 脱开后，由于第三对轮齿还未进入啮合，所以，此时只有一对轮齿相啮合
 - 当第二对轮齿到达 K 时，第三对轮齿又进入点 B_2 ，随之又是两对轮齿相啮合
 - 如此不断循环下去

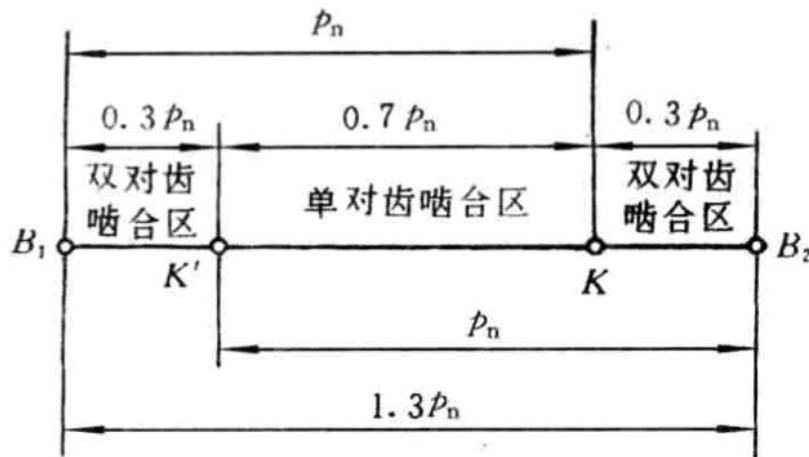


图 4-10 重合度的物理意义

- 所以， $\overline{B_1K}$ 和 $\overline{B_2K'}$ 为双对轮齿啮合区，其长度各为 $0.3p_n$ ；而 $\overline{KK'}$ 为单对轮齿啮合区，其长度为 $0.7p_n$
- 从理论上分析，只要重合度 $\varepsilon_a = 1$ 就能保证一对齿轮的连续定角速比啮合
 - 但因齿轮的制造、安装不可避免地会有误差，为了确保一对齿轮连续定角速比啮合，应使所设计的一对齿轮的重合度 $\varepsilon_a > 1$
 - 在实际应用中，根据不同的情况，应使 $\varepsilon_a \geq [\varepsilon_a]$ 许用值

使用场合	一般机械	汽车、拖拉机	金属切削机床
$[\varepsilon_a]$	1.4	1.1~1.2	1.3

渐开线直齿圆柱齿轮机构的啮合传动

• 6. 连续传动条件：3)重合度与基本参数的关系

• 如图所示， $\overline{B_1B_2} = \overline{B_1P} + \overline{PB_2}$ ，而

$$\bullet \quad \overline{B_1P} = \overline{B_1N_1} - \overline{PN_1} = \frac{mz_1}{2} \cos \alpha (\tan \alpha_{a1} - \tan \alpha')$$

$$\bullet \quad \overline{B_2P} = \overline{B_2N_2} - \overline{PN_2} = \frac{mz_2}{2} \cos \alpha (\tan \alpha_{a2} - \tan \alpha')$$

• 将 $\overline{B_1B_2} = \overline{B_1P} + \overline{PB_2}$ 代入 $\varepsilon_a = \frac{\overline{B_1B_2}}{p_n} \geq 1$ ，可得

$$\bullet \quad \varepsilon_a = \frac{\overline{B_1B_2}}{p_n} = \frac{1}{2\pi} [z_1(\tan \alpha_{a1} - \tan \alpha') + z_2(\tan \alpha_{a2} - \tan \alpha')]$$

• α_{a1} 、 α_{a2} 分别为齿轮 1、2 的齿顶圆压力角，其值为

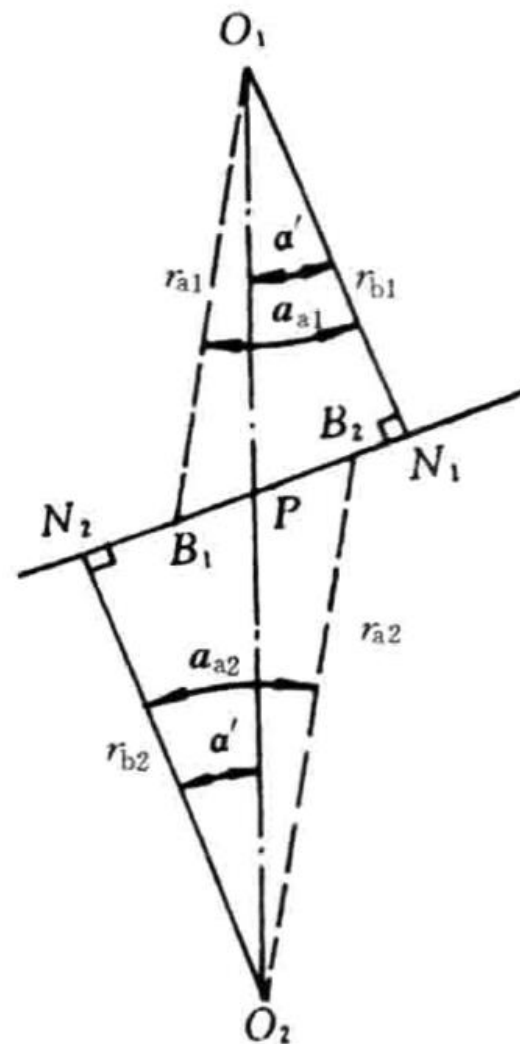
$$\bullet \quad \cos \alpha_{a1} = r_{b1}/r_{a1}, \quad \cos \alpha_{a2} = r_{b2}/r_{a2}$$

• 直齿圆柱齿轮在啮合传动中，不可能保证总是有两对齿啮合，因而限制了直齿圆柱齿轮机构的承载能力

• 由上述可知，一对标准直齿圆柱齿轮传动在满足无侧隙啮合条件的情况下，重合度 ε_a 与模数无关

• 而齿数增多，重合度增大

• 当齿数 z 趋向无穷多（即齿轮变为齿条）时，可求出直齿圆柱齿轮传动重合度的极限值 $\varepsilon_{amax} = 1.981$



怎么推导呢？

渐开线齿廓的切削加工原理

- 齿轮轮齿加工的方法很多，可分为
 - 仿形法——刀具切削刃的形状，在其轴向剖面内与被切齿轮齿槽的形状相同
 - 范成法——利用齿轮啮合原理加工齿廓

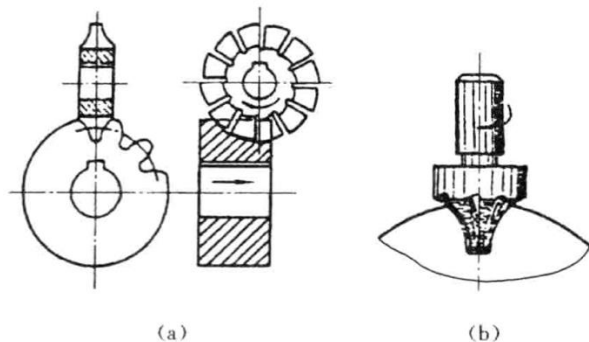


图 4-12 渐开线齿廓的仿形法加工

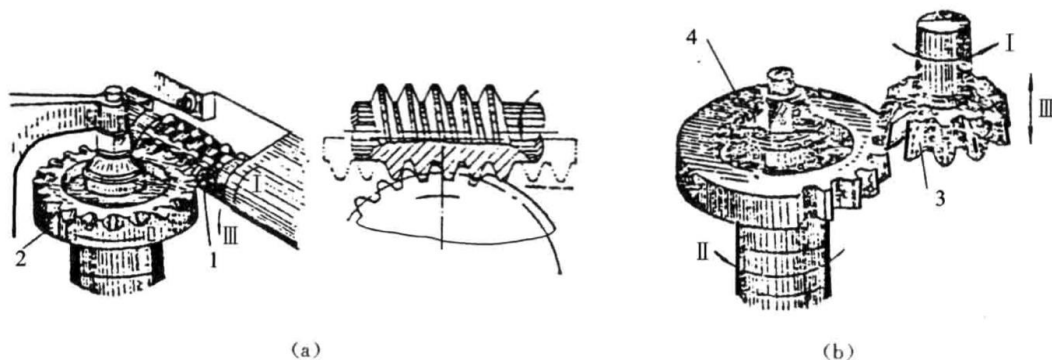
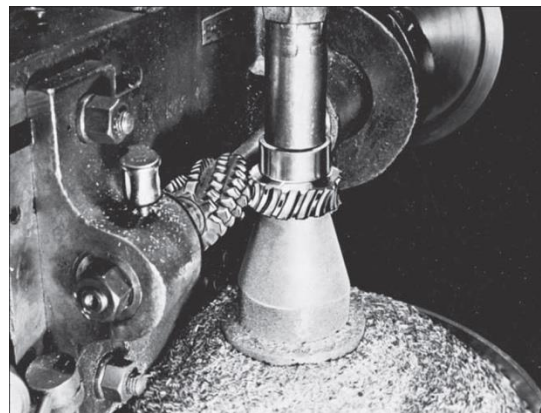


图 4-13 渐开线齿廓的范成法加工

1—右旋滚刀；2、4—被切齿轮；3—齿轮插刀



The Forming of Gear Teeth



The Making of the Spur Gears

Raw Materials



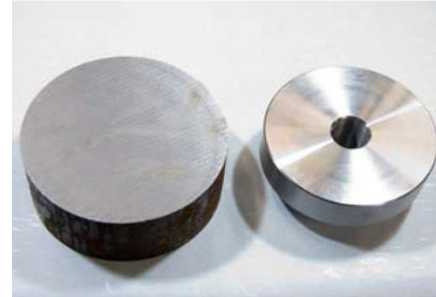
Raw materials bought from material makers are kept in stock. The materials are six meters in length.

Sawing



The materials are cut to size.

Shaping



With a lathe, a cut workpiece is shaped into gear blank.

Tooth-Cutting



Tooth-cutting has been done with a gear hobbing machine. The cutting usually leaves burrs on the teeth.

Deburring



Rough spots on the teeth have been smoothed with a deburring machine.

Black Oxide Finish



The black oxide finish is somewhat effective in preventing rust.

Packaging



To ensure delivery in good condition each and every spur gear is individually packaged.

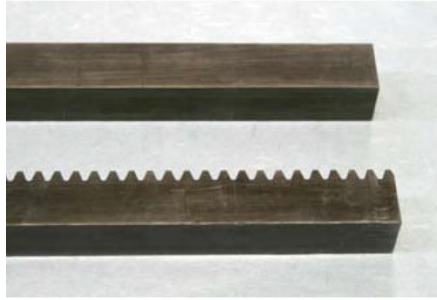
The Making of the Bevel Gears

Raw Materials



Raw Materials bought from material makers are kept in stock.

Tooth-Cutting



Tooth-cutting has been done with a rack cutting machine. The cutting usually leaves burrs on the teeth.

Deburring



Rough spots on the teeth have been smoothed with a deburring machine.

Straightening



To straighten warping, pressure is applied on racks with a hydraulic press

Machining Ends



Both ends have been machined so that racks can be butted against each other to make any desired length.

Processing holes



Mounting screw holes have been drilled for easier assembly.

Black Oxide Finish



The black oxide finish is somewhat effective in preventing rust.

Packaging



To ensure delivery in good condition each and every rack is individually packaged.

The Making of the Racks

Raw materials



Raw materials bought from material makers are kept in stock. The materials are six meters in length.

Sawing



The materials are cut to size.

Shaping



With a lathe, a cut workpiece is shaped into gear blank.

Tooth-Cutting



Tooth-cutting has been done with a Coniflex generator. The cutting usually leaves burrs on the teeth.

Deburring



Rough spots on the teeth are being smoothed with a deburring machine.

Black Oxide Finish



The black oxide finish is somewhat effective in preventing rust.

Packaging



To ensure delivery in good condition each and every bevel gear is individually packaged.

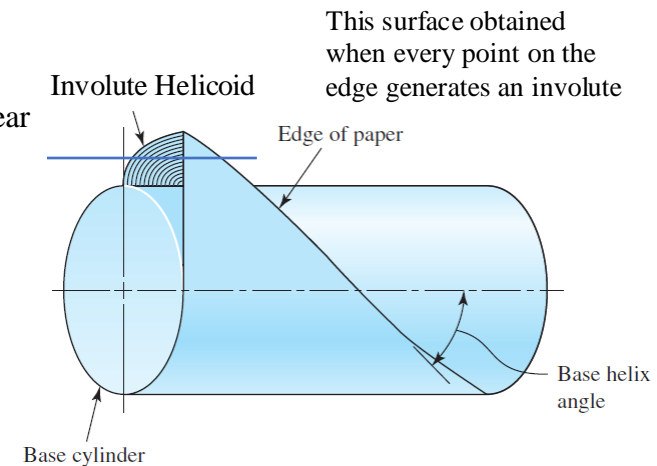
其他齿轮机构 及其传动分析

齿轮机构及其设计

Parallel Helical Gears



- The helix angle is the same on each gear, but one gear must have a right-hand helix and the other a left-hand helix.
- Gradual engagement of the teeth
 - The initial contact is a point that extends into a line as the teeth come into more engagement
 - Enables a smooth transfer of load from one tooth to another
 - The ability to transmit heavy loads at high speeds
 - **Contact Ratio** is of only minor importance
 - **Contact Area** becomes significant, proportional to the face width of the gear
- Loading Types: Radial & Thrust
 - Minimum thrust load as a design preference
- Double Helical Gears
 - When the thrust loads become high
 - Equivalent to two helical gears of opposite hand, mounted side by side on the same shaft
 - Develop opposite thrust reactions and thus cancel out the thrust load.



Example on Helical Gear

A stock helical gear has a normal pressure angle of 20° , a helix angle of 25° , and a transverse diametral pitch of 6 teeth/in, and has 18 teeth. Find:

- The pitch diameter
- The transverse, the normal, and the axial pitches
- The normal diametral pitch
- The transverse pressure angle

$$d = \frac{N}{P_t} = \frac{18}{6} = 3 \text{ in}$$

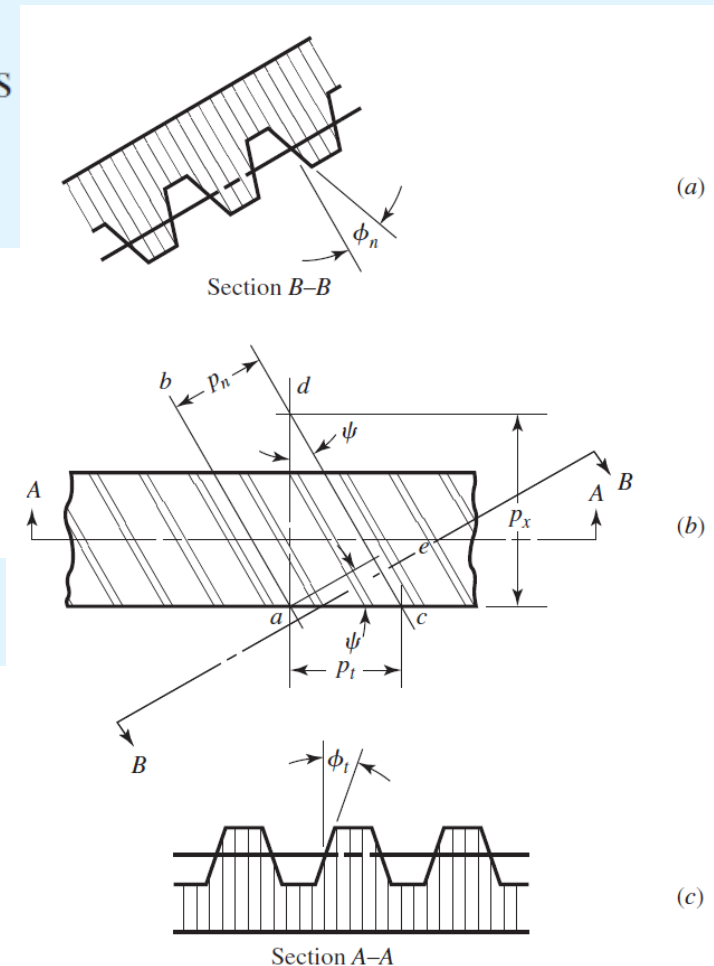
$$P_n = \frac{P_t}{\cos \psi} = \frac{6}{\cos 25^\circ} = 6.620 \text{ teeth/in}$$

$$p_t = \frac{\pi}{P_t} = \frac{\pi}{6} = 0.5236 \text{ in}$$

$$\phi_t = \tan^{-1} \left(\frac{\tan \phi_n}{\cos \psi} \right) = \tan^{-1} \left(\frac{\tan 20^\circ}{\cos 25^\circ} \right) = 21.88^\circ$$

$$p_n = p_t \cos \psi = 0.5236 \cos 25^\circ = 0.4745 \text{ in}$$

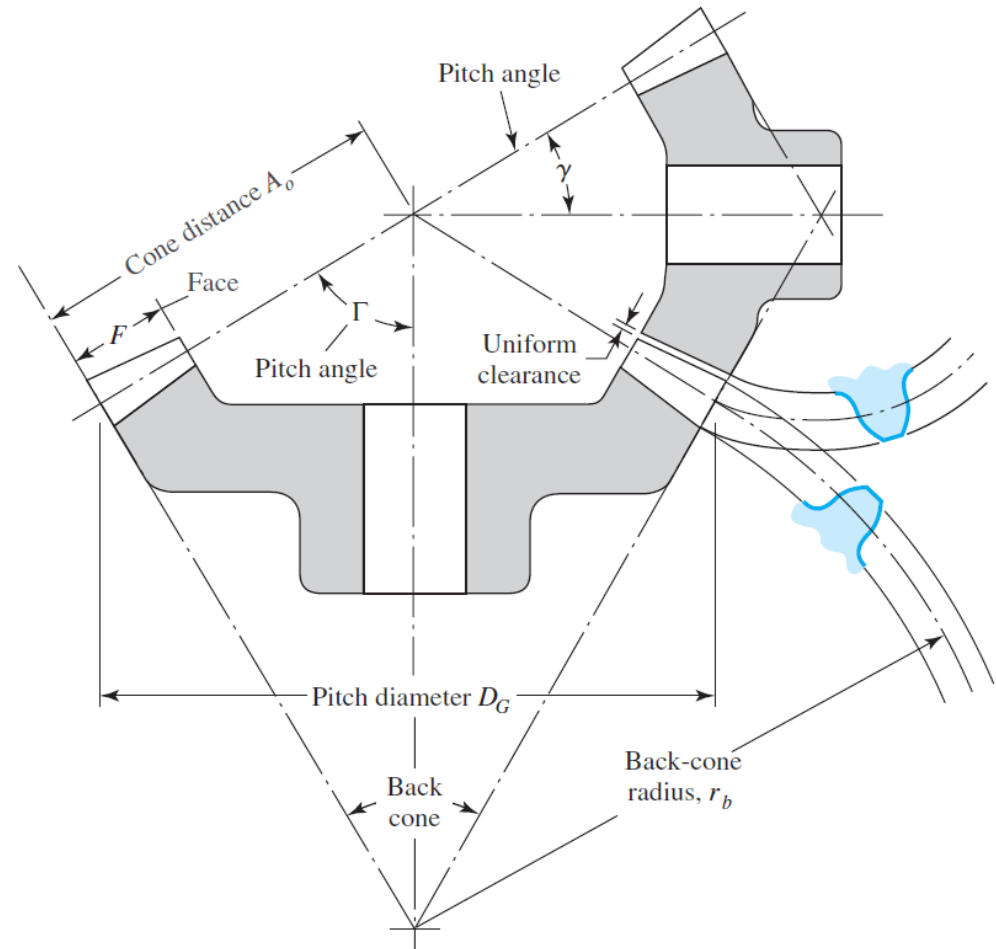
$$p_x = \frac{p_t}{\tan \psi} = \frac{0.5236}{\tan 25^\circ} = 1.123 \text{ in}$$



Straight Bevel Gears

- The pitch of bevel gears
 - measured at the large end of the tooth
 - both the circular pitch and the pitch diameter are calculated in the same manner as for spur gears
- Pitch Angles
 - Defined by the pitch cones meeting at the apex

$$\tan \gamma = \frac{N_P}{N_G} \quad \tan \Gamma = \frac{N_G}{N_P}$$



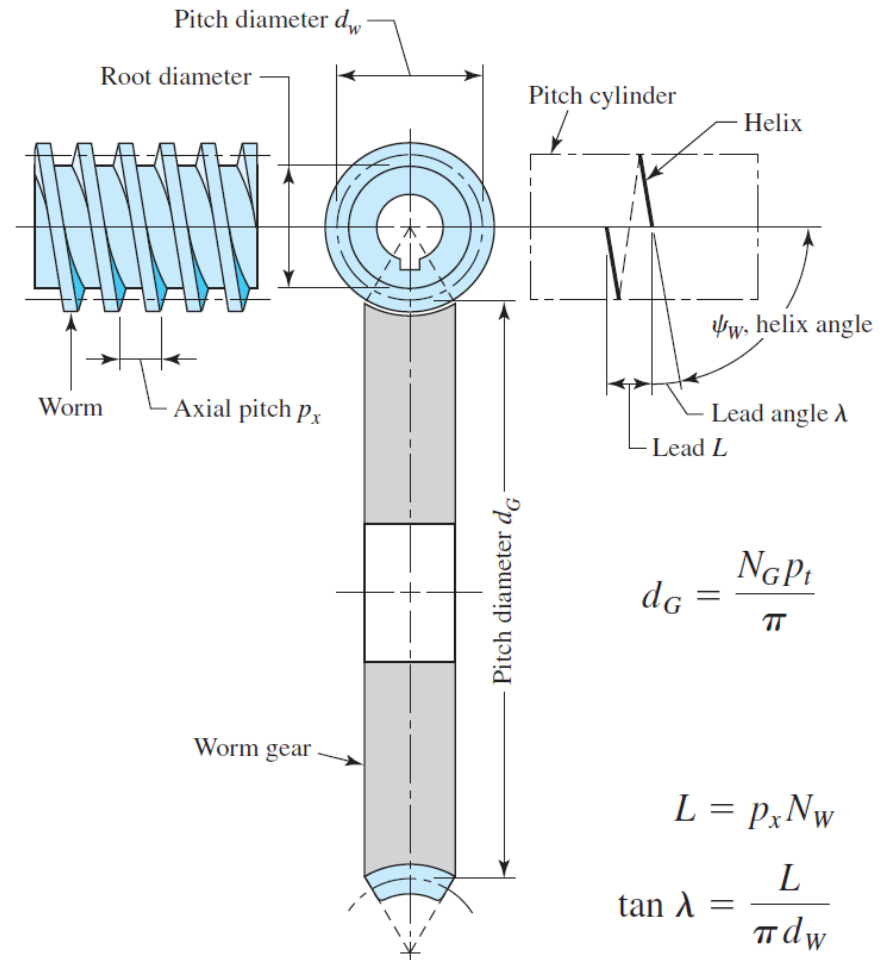
Worm Gears

- The worm and worm gear of a set have the same hand of helix as for crossed helical gears, but the helix angles are usually quite different.
 - The *helix angle* on the **worm** is generally quite large
 - thus *lead angle* λ on the **worm** is often used
 - The *helix angle* ψ_G on the **gear** very small
 - $\lambda = \psi_G$ for a 90° shaft angle.

- Pitch of the Worm Gearsets
 - p_x : Axial Pitch of the worm
 - p_t : Transverse Circular Pitch of the mating gear
 - $p_x = p_t$ for a 90° shaft angle

- Generally, the pitch diameter of the worm should be selected so as to fall into the range

$$\frac{C^{0.875}}{3.0} \leq d_w \leq \frac{C^{0.875}}{1.7} \quad C \text{ is the center distance}$$



$$d_G = \frac{N_G p_t}{\pi}$$

$$L = p_x N_w$$

$$\tan \lambda = \frac{L}{\pi d_w}$$

Example on Gear Fundamentals

A gearset consists of a 16-tooth pinion driving a 40-tooth gear. The diametral pitch is 2, and the addendum and dedendum are $1/P$ and $1.25/P$, respectively. The gears are cut using a pressure angle of 20° .

(a) Compute the circular pitch, the center distance, and the radii of the base circles.

$$(a) \quad p = \frac{\pi}{P} = \frac{\pi}{2} = 1.571 \text{ in}$$

The pitch diameters of the pinion and gear are, respectively,

$$d_p = \frac{N_p}{P} = \frac{16}{2} = 8 \text{ in} \quad d_G = \frac{N_G}{P} = \frac{40}{2} = 20 \text{ in}$$

Therefore the center distance is

$$\frac{d_p + d_G}{2} = \frac{8 + 20}{2} = 14 \text{ in}$$

Since the teeth were cut on the 20° pressure angle, the base-circle radii are found to be, using $r_b = r \cos \phi$,

$$r_b(\text{pinion}) = \frac{8}{2} \cos 20^\circ = 3.759 \text{ in}$$

$$r_b(\text{gear}) = \frac{20}{2} \cos 20^\circ = 9.397 \text{ in}$$

Example on Gear Fundamentals

A gearset consists of a 16-tooth pinion driving a 40-tooth gear. The diametral pitch is 2, and the addendum and dedendum are $1/P$ and $1.25/P$, respectively. The gears are cut using a pressure angle of 20° .

(b) In mounting these gears, the center distance was incorrectly made $\frac{1}{4}$ in larger. Compute the new values of the pressure angle and the pitch-circle diameters.

(b) Designating d'_P and d'_G as the new pitch-circle diameters, the $\frac{1}{4}$ -in increase in the center distance requires that

$$\frac{d'_P + d'_G}{2} = 14.250 \quad (1)$$

Also, the velocity ratio does not change, and hence

$$\frac{d'_P}{d'_G} = \frac{16}{40} \quad (2)$$

Solving Eqs. (1) and (2) simultaneously yields

$$d'_P = 8.143 \text{ in} \quad d'_G = 20.357 \text{ in}$$

Since $r_b = r \cos \phi$, using either the pinion or gear, the new pressure angle is

$$\phi' = \cos^{-1} \frac{r_b(\text{pinion})}{d'_P/2} = \cos^{-1} \frac{3.759}{8.143/2} = 22.59^\circ$$

Gear Trains

- Consider a pinion 2 driving a gear 3. The speed of the driven gear is

$$n_3 = \left| \frac{N_2}{N_3} n_2 \right| = \left| \frac{d_2}{d_3} n_2 \right|$$

where n = revolutions or rev/min
 N = number of teeth
 d = pitch diameter

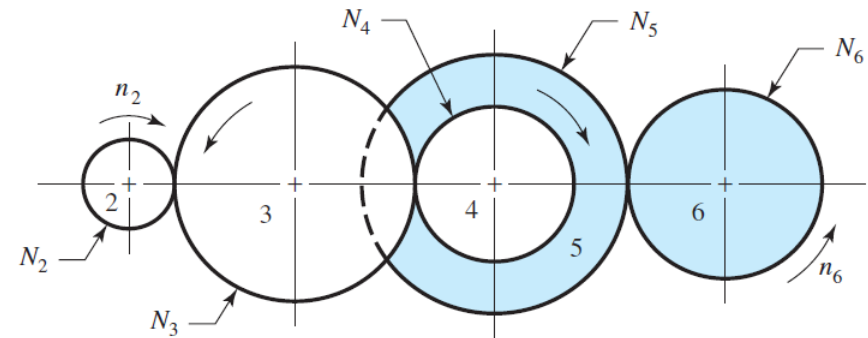
The absolute-value signs are used to permit complete freedom in choosing positive and negative directions.

- Train Value

$$e = \frac{\text{product of driving tooth numbers}}{\text{product of driven tooth numbers}}$$

$$n_L = e n_F$$

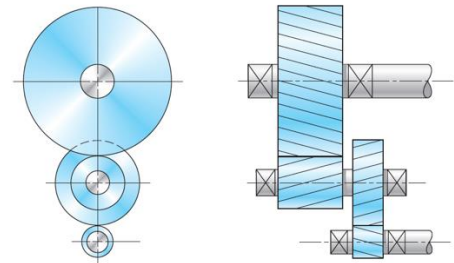
where n_L is the speed of the last gear in the train and n_F is the speed of the first.



$$n_7 = -\frac{N_2}{N_3} \frac{N_3}{N_4} \frac{N_5}{N_6} n_2$$

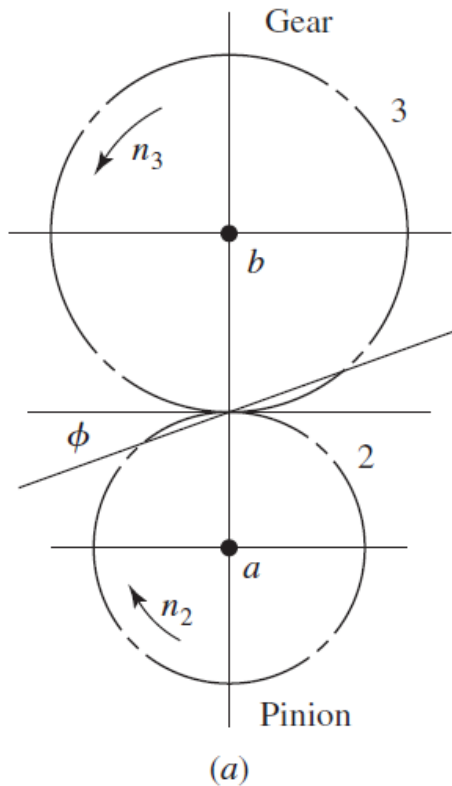
- As a rough guideline,

- A train value of up to 10 to 1 can be obtained with one pair of gears.
- Greater ratios can be obtained in less space and with fewer dynamic problems by compounding additional pairs of gears.
- A two-stage compound gear train can obtain a train value of up to 100 to 1.



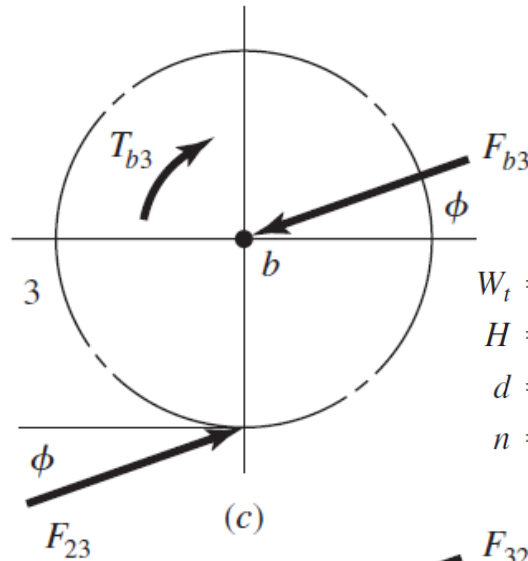
Force Analysis – Spur Gearing

- 1: frame of the machine
- 2: input gear
- 3, 4, etc.: gears next
- a, b, c, etc.: shafts
- F_{xy} : forced exerted by *x* against *y*
- r/t*: radial or tangential components

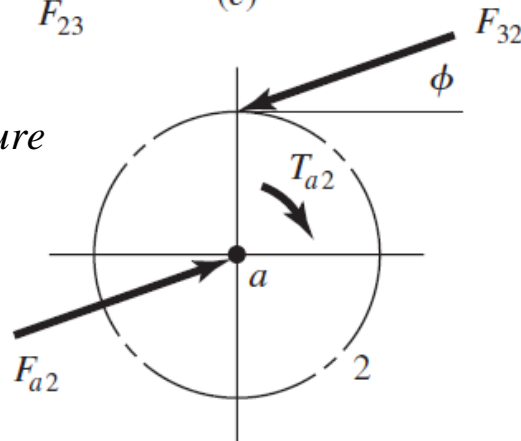


(a)

the pressure line



(c)



(b)

transmitted load $W_t = F_{32}^t$

applied torque $T = \frac{d}{2} W_t$

transmitted power $H = T\omega = (W_t d/2)\omega$

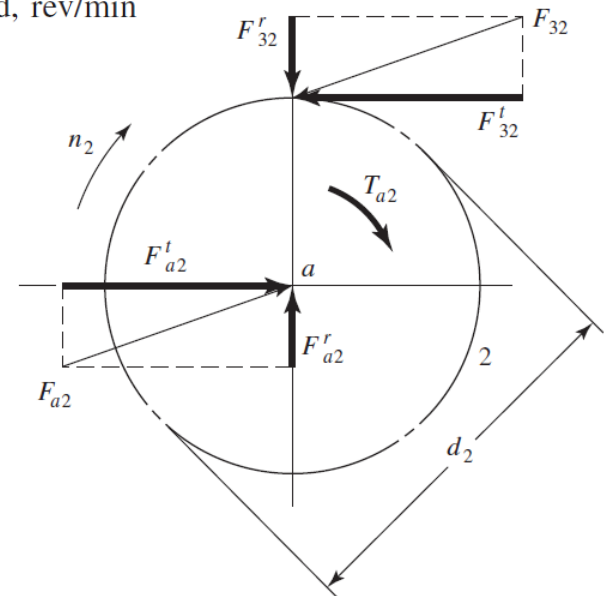
W_t = transmitted load, kN

H = power, kW

d = gear diameter, mm

n = speed, rev/min

$$W_t = \frac{60\,000 H}{\pi d n}$$



Pinion 2 runs at 1750 rev/min and transmits 2.5 kW to idler gear 3. The teeth are cut on the 20° full-depth system and have a module of $m = 2.5$ mm. Analyze the forces on gear 3.

- The pitch diameters

$$d_2 = N_2 m = 20(2.5) = 50 \text{ mm}$$

$$d_3 = N_3 m = 50(2.5) = 125 \text{ mm}$$

- The transmitted load

$$W_t = \frac{60\,000H}{\pi d_2 n} = \frac{60\,000(2.5)}{\pi(50)(1750)} = 0.546 \text{ kN}$$

- The tangential force of gear 2 on gear 3

$$F_{23}^r = F_{23}^t \tan 20^\circ = (0.546) \tan 20^\circ = 0.199 \text{ kN}$$

The resultant shaft reaction

$$F_{b3} = \sqrt{(0.347)^2 + (0.347)^2} = 0.491 \text{ kN}$$

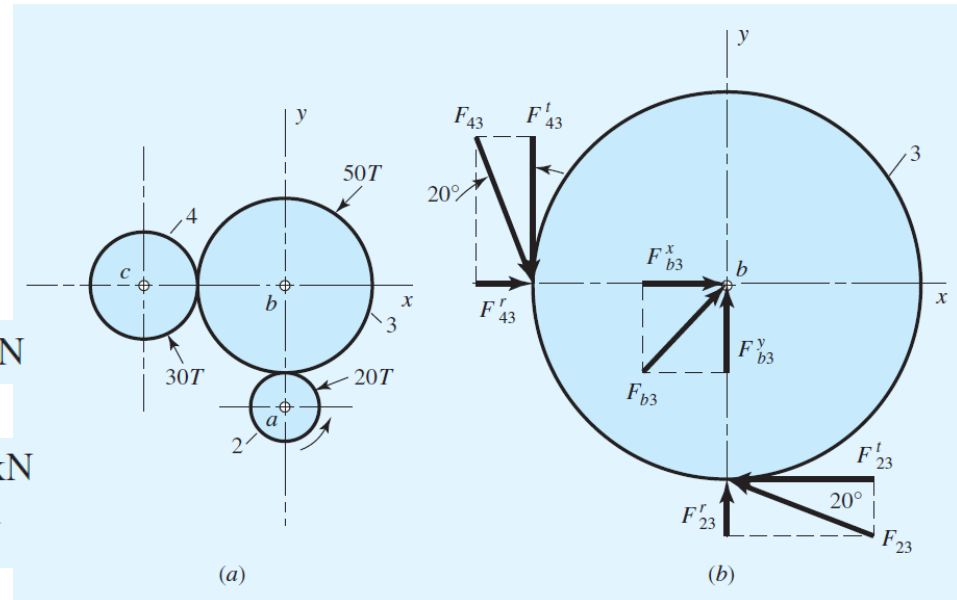
$$F_{23} = \frac{F_{23}^t}{\cos 20^\circ} = \frac{0.546}{\cos 20^\circ} = 0.581 \text{ kN}$$

- Idler gear transmits no power to its shaft, so the tangential reaction of gear 4 on gear 3 is also equal to the transmitted load*

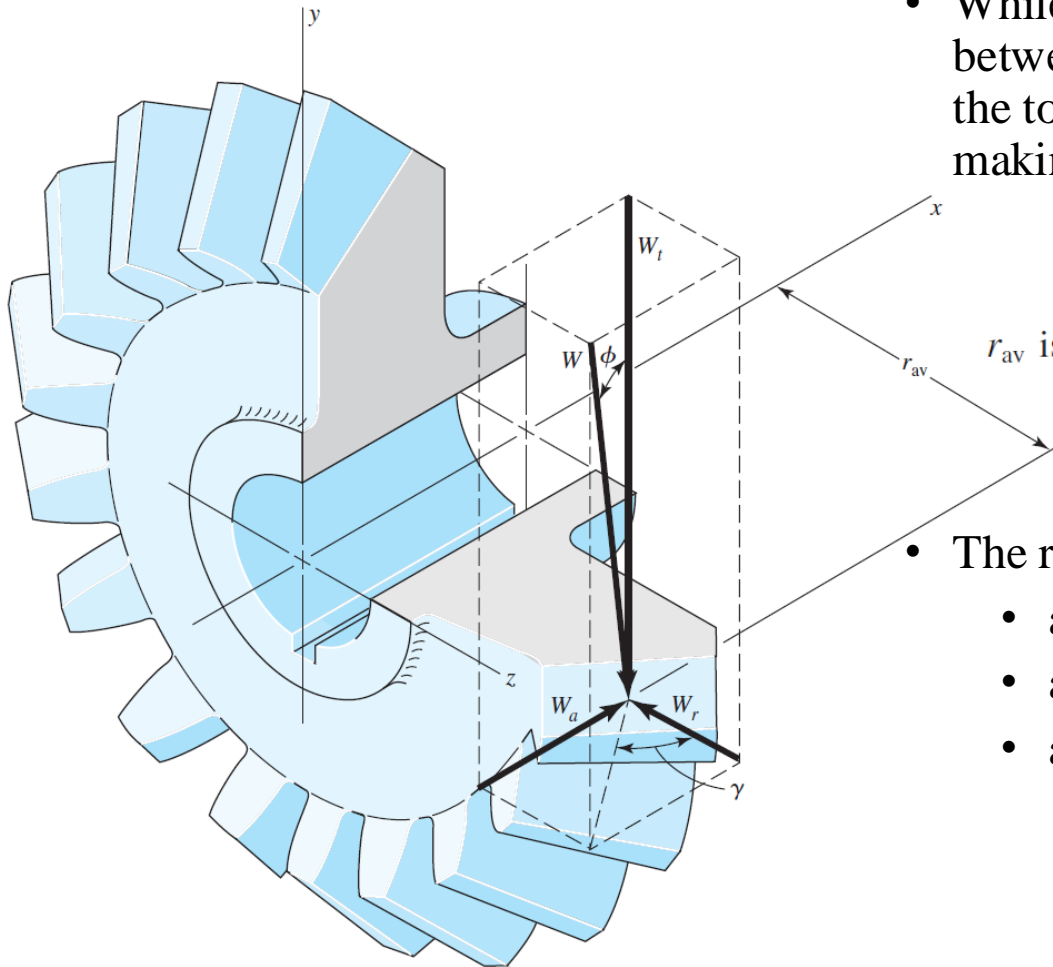
$$F_{43}^t = 0.546 \text{ kN} \quad F_{43}^r = 0.199 \text{ kN} \quad F_{43} = 0.581 \text{ kN}$$

$$F_{b3}^x = -(F_{23}^t + F_{43}^r) = -(-0.546 + 0.199) = 0.347 \text{ kN}$$

$$F_{b3}^y = -(F_{23}^r + F_{43}^t) = -(0.199 - 0.546) = 0.347 \text{ kN}$$



Force Analysis - Bevel Gearing



- While the actual resultant occurs somewhere between the midpoint and the large end of the tooth, there is only a small error in making this assumption.

$$W_t = \frac{T}{r_{av}}$$

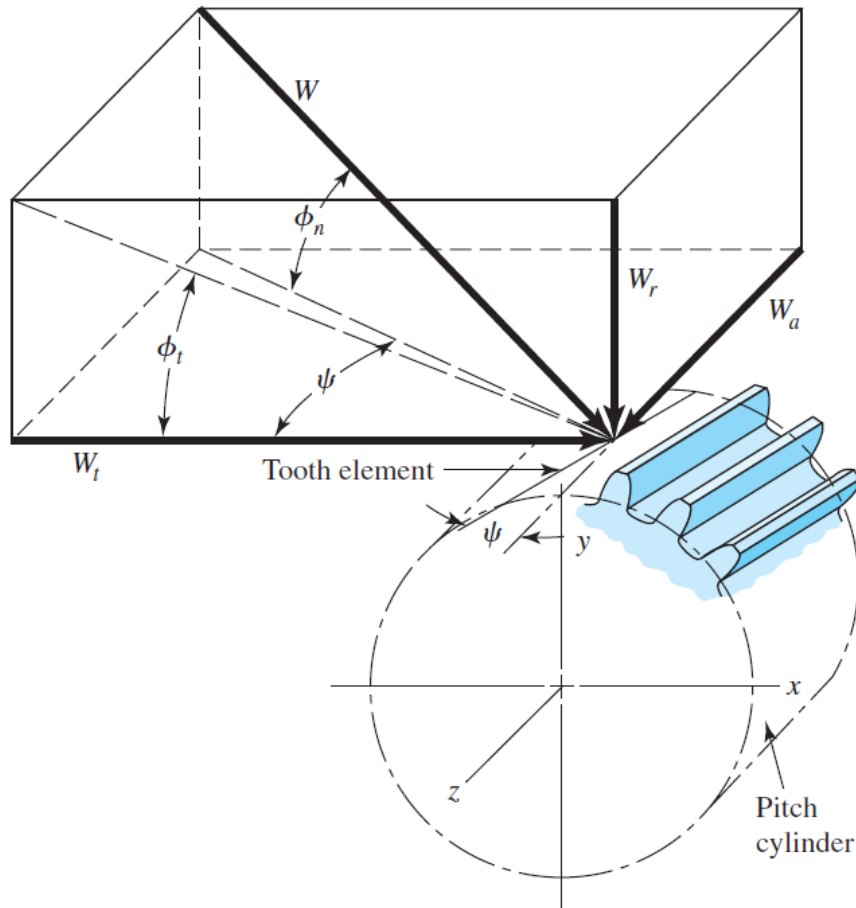
r_{av} is the pitch radius at the midpoint of the tooth

- The resultant force W has three components:
 - a tangential force,
 - a radial force, and
 - an axial force.

$$W_r = W_t \tan \phi \cos \gamma$$

$$W_a = W_t \tan \phi \sin \gamma$$

Force Analysis - Helical Gearing



- The three components of the total (normal) tooth force W are

$$W_r = W \sin \phi_n$$

$$W_t = W \cos \phi_n \cos \psi$$

$$W_a = W \cos \phi_n \sin \psi$$

W = total force

W_r = radial component

W_t = tangential component, also called the transmitted load

W_a = axial component, also called the thrust load

- Usually W_t is given and the other forces are desired

$$W_r = W_t \tan \phi_t$$

$$W_a = W_t \tan \psi$$

$$W = \frac{W_t}{\cos \phi_n \cos \psi}$$

Example – Helical Gearing

An electric motor transmits 1-hp at 1800 rev/min in the clockwise direction, as viewed from the positive x axis. Keyed to the motor shaft is an 18-tooth helical pinion having a normal pressure angle of 20° , a helix angle of 30° , and a normal diametral pitch of 12 teeth/in.

Make a three-dimensional sketch of the motor shaft and pinion, and show the forces acting on the pinion and the bearing reactions at A and B . The thrust should be taken out at A .

- The transverse pressure angle

$$\phi_t = \tan^{-1} \frac{\tan \phi_n}{\cos \psi} = \tan^{-1} \frac{\tan 20^\circ}{\cos 30^\circ} = 22.8^\circ$$

- The transverse pitch

$$P_t = P_n \cos \psi = 12 \cos 30^\circ = 10.39 \text{ teeth/in.}$$

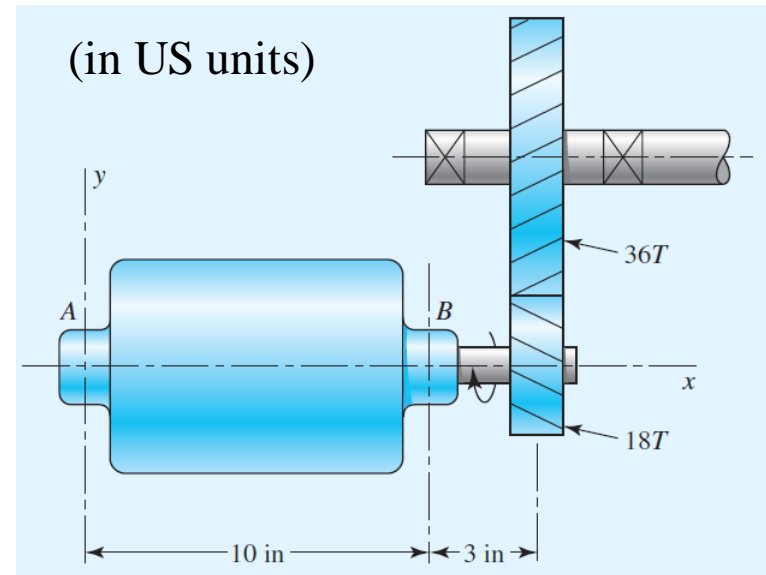
- Therefore the pitch diameter of the pinion

$$d_p = 18/10.39 = 1.732 \text{ in}$$

- The pitch-line velocity

$$V = \frac{\pi d n}{12} = \frac{\pi (1.732)(1800)}{12} = 816 \text{ ft/min}$$

- The transmitted load $W_t = \frac{33\,000H}{V} = 40.4 \text{ lbf}$

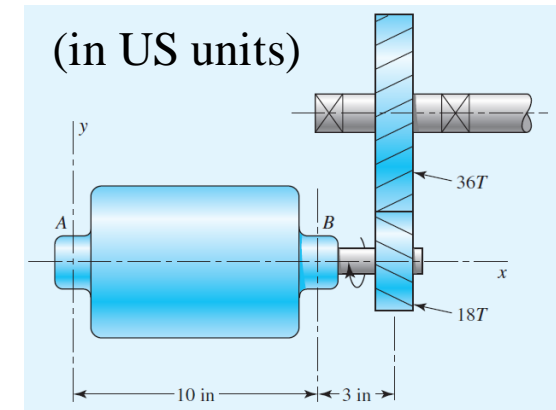
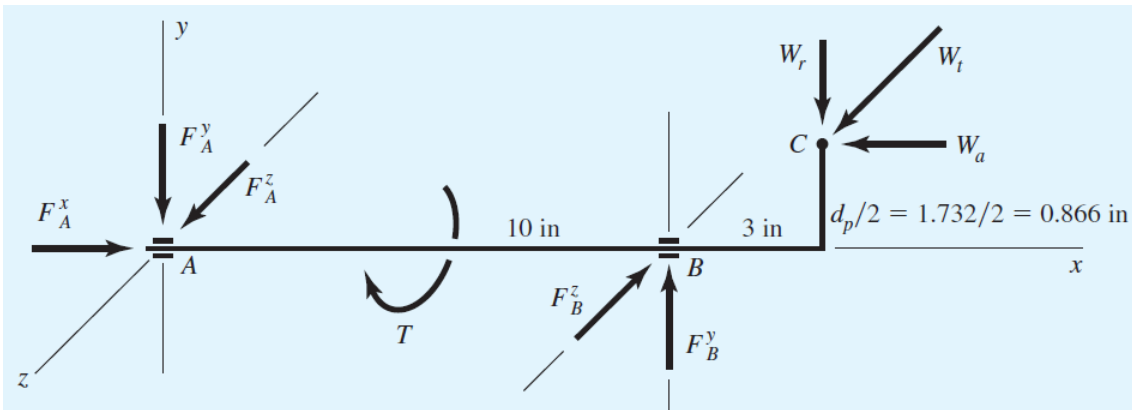


$$W_r = W_t \tan \phi_t = (40.4) \tan 22.8^\circ = 17.0 \text{ lbf}$$

$$W_a = W_t \tan \psi = (40.4) \tan 30^\circ = 23.3 \text{ lbf}$$

$$W = \frac{W_t}{\cos \phi_n \cos \psi} = \frac{40.4}{\cos 20^\circ \cos 30^\circ} = 49.6 \text{ lbf}$$

Example – Helical Gearing



- We assume bearing reactions at A and B as above: $F_A^x = W_a = 23.3 \text{ lbf.}$
- Taking moments about the z axis: $-(17.0)(13) + (23.3)(0.866) + 10F_B^y = 0$ $F_B^y = 20.1 \text{ lbf.}$
- Summing forces in the y direction: $F_A^y = 3.1 \text{ lbf.}$
- Taking moments about the y axis: $10F_B^z - (40.4)(13) = 0$ $F_B^z = 52.5 \text{ lbf.}$
- Summing forces in the z direction: $F_A^z = 12.1 \text{ lbf.}$
- The torque: $T = W_t d_p/2 = (40.4)(1.732/2) = 35 \text{ lbf} \cdot \text{in}$
- *Can be alternatively solved by using vector method (refer to the textbook)*

Force Analysis - Worm Gearing

If friction is neglected, then the only force exerted by the gear will be the force W .

$$W^x = W \cos \phi_n \sin \lambda$$

$$W^y = W \sin \phi_n$$

$$W^z = W \cos \phi_n \cos \lambda$$

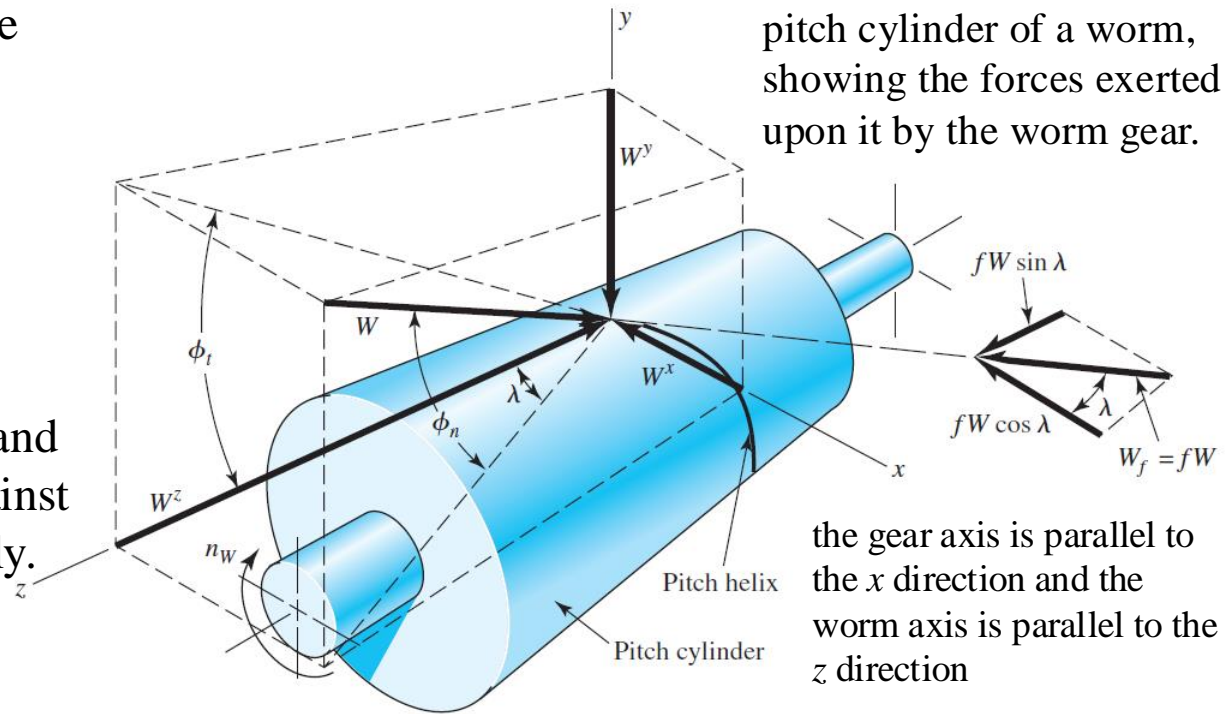
We now use the subscripts W and G to indicate forces acting against the worm and gear, respectively.

$$W_{Wt} = -W_{Ga} = W^x$$

$$W_{Wr} = -W_{Gr} = W^y$$

$$W_{Wa} = -W_{Gt} = W^z$$

The force W acting normal to the worm-tooth profile produces a frictional force $W_f = fW$.



$$W^x = W(\cos \phi_n \sin \lambda + f \cos \lambda)$$

$$W^y = W \sin \phi_n$$

$$W^z = W(\cos \phi_n \cos \lambda - f \sin \lambda)$$

$$W_f = fW = \frac{f W_{Gt}}{f \sin \lambda - \cos \phi_n \cos \lambda}$$

Friction & Sliding of the Worm Gearing

- Efficiency $\eta = \frac{W_{wt} \text{ (without friction)}}{W_{wt} \text{ (with friction)}}$
- A useful relation between the two tangential forces

$$\eta = \frac{\cos \phi_n - f \tan \lambda}{\cos \phi_n + f \cot \lambda}$$

↑ $f = 0$

$$W_{Wt} = -W_{Ga} = W^x \quad \color{teal}{\rule{1em}{1em}} \quad W^x = W(\cos \phi_n \sin \lambda + f \cos \lambda)$$

$$W_{Wr} = -W_{Gr} = W^y \quad \quad \quad W^y = W \sin \phi_n$$

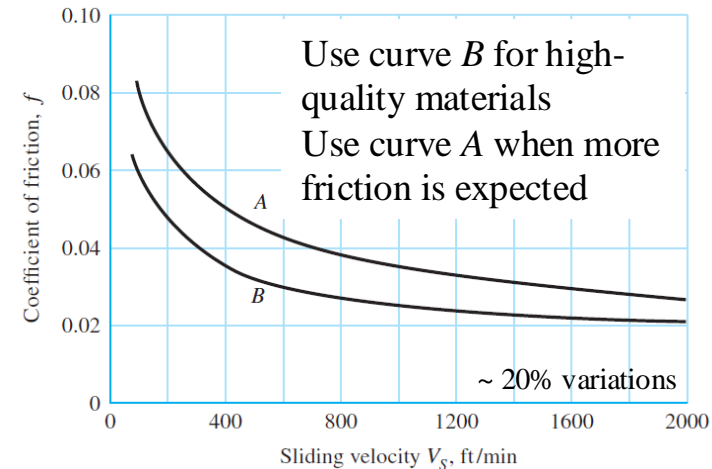
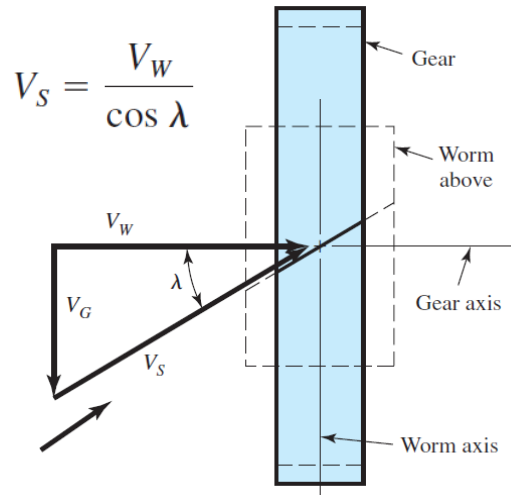
$$W_{Wa} = -W_{Gt} = W^z \quad \color{teal}{\rule{1em}{1em}} \quad W^z = W(\cos \phi_n \cos \lambda - f \sin \lambda)$$

$$W_{Wt} = W_{Gt} \frac{\cos \phi_n \sin \lambda + f \cos \lambda}{f \sin \lambda - \cos \phi_n \cos \lambda}$$

Efficiency of Worm Gearsets for $f = 0.05$

Lead Angle λ , deg	Efficiency η , %
1.0	25.2
2.5	45.7
5.0	62.6
7.5	71.3
10.0	76.6
15.0	82.7
20.0	85.6
30.0	88.7

Many experiments have shown that the coefficient of friction is dependent on the relative or *sliding velocity*.





机械设计

Design & Learning Research Group

谢谢~

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