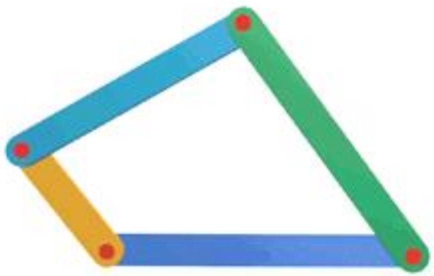
第02章
平面连杆机构

第01节 四连杆机构

宋超阳
songcy@ieee.org

本章要点概述

- 平面四杆机构的基本形式、演变及其应用
- 平面四杆机构设计中的共性问题
- 平面四杆机构的设计
- 平面连杆机构的解析综合



Kinematics

- Degree of Freedom
  - *The number of coordinates needed to define its position in space*
    - One-DOF Mechanism: DOF = 1
    - Multi-DOF Mechanism: DOF > 1
    - Structure: DOF = 0
    - Preloaded Structure: DOF < 1

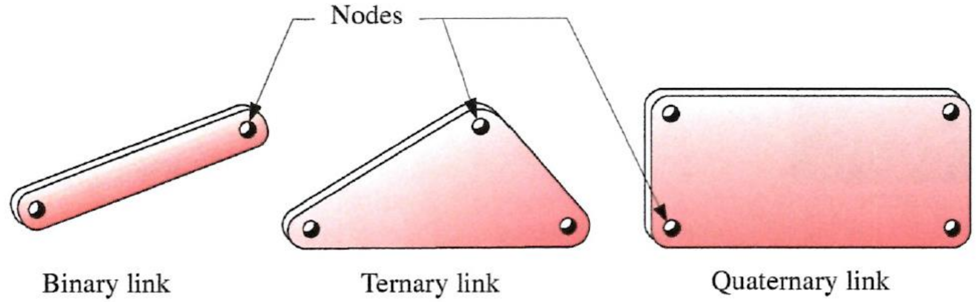
- Mechanisms: variants of a linkage
  - *A collection of* ***links*** *and* ***joints****, one of which is grounded, and all are interconnected in a way to provide a controlled output in response to one or more inputs.*

Links, Joints & Mobility

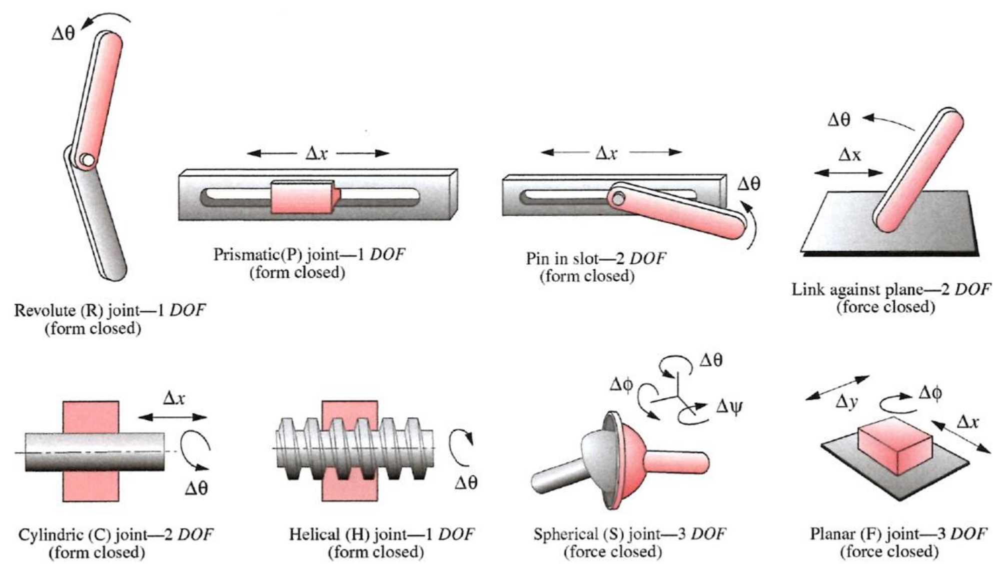
- Link
  - A rigid body of any shape that has some number of attachment points called nodes, that allow multiple links to be connected by joints
- Joints
  - Characterized by their geometry, by the number of DOF they allow between the links they join, and by whether they are held together (closed) by a ***force*** or by their ***form*** (geometry)
- Kutzbach Equation

$$M = 3(L-1) - 2J_1 - J_2$$

  - M: Mobility (DOF)
  - L: the number of links
  - J_1: the number of one-DOF joints
  - J_2: the number of two-DOF joints
  - ***[Doesn't really work]***



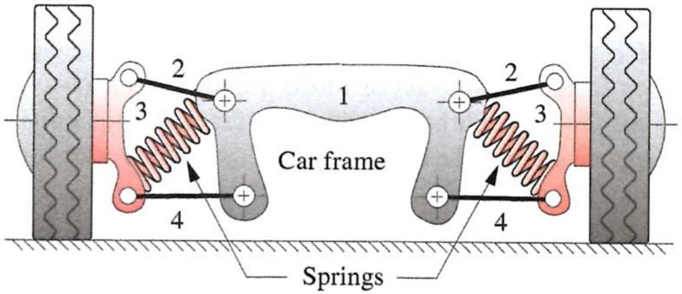
(a) Some links—their names reflect the number of nodes



(b) Some joint types—note their DOF and type of closure

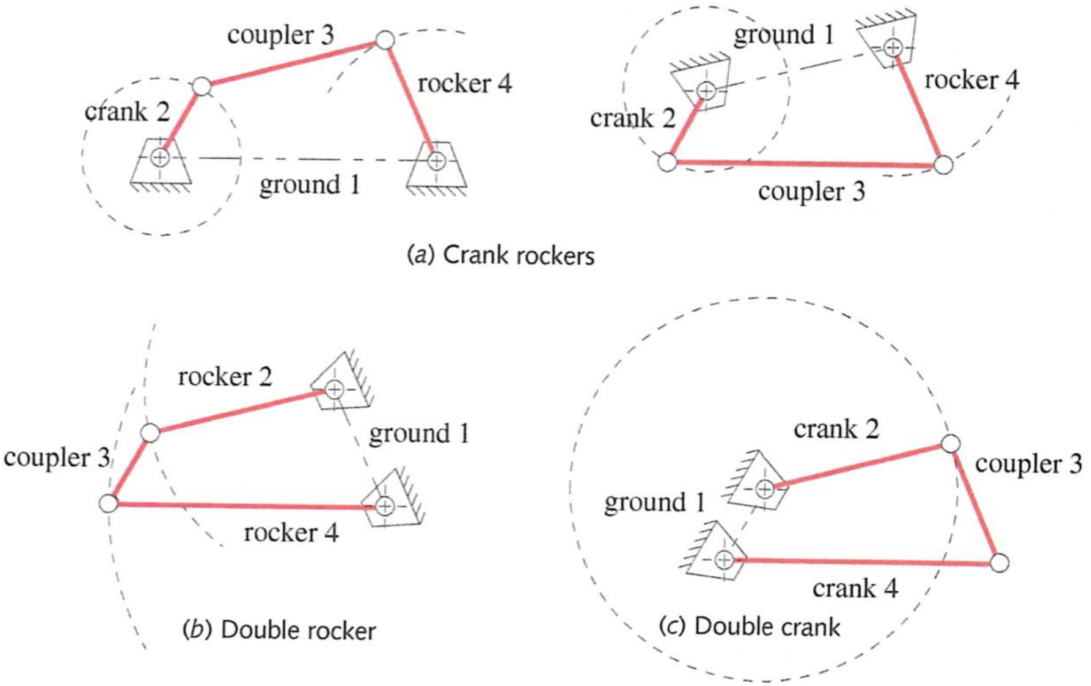
The Fourbar Linkage

- Grashof Criterion $S + L \leq P + Q$
  - S / L: The length of the shortest / longest link
  - P & Q: The lengths of the other two



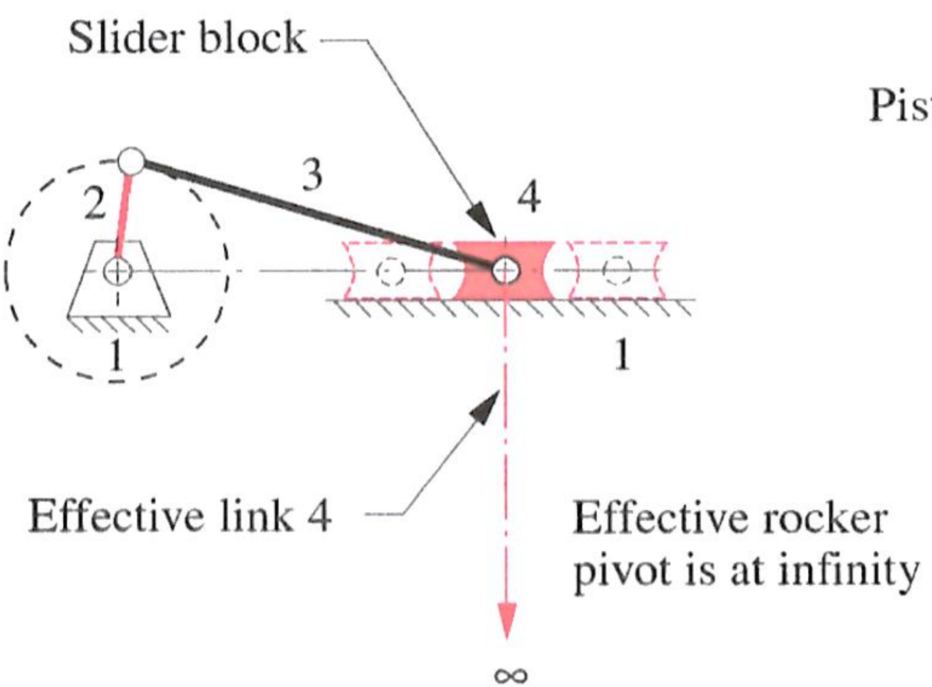
- A Grashof Linkage
  - At least one link can revolve fully
  - It has "change-point" positions when equal [Singularity]

Fourbar linkage is simply the best linkage of all.

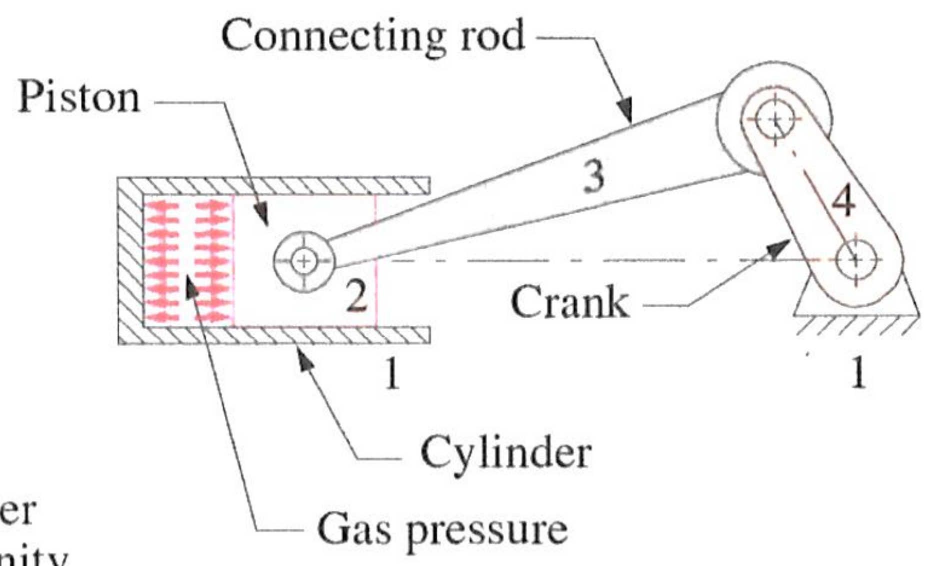


Fourbar Crank-Slider & Slider-Crank

- *If a rocker of a fourbar crank rocker linkage is increased in length indefinitely, it effectively becomes infinite in length and the linkage is transformed into a fourbar crank-slider, which is similar for a fourbar slider-crank.*



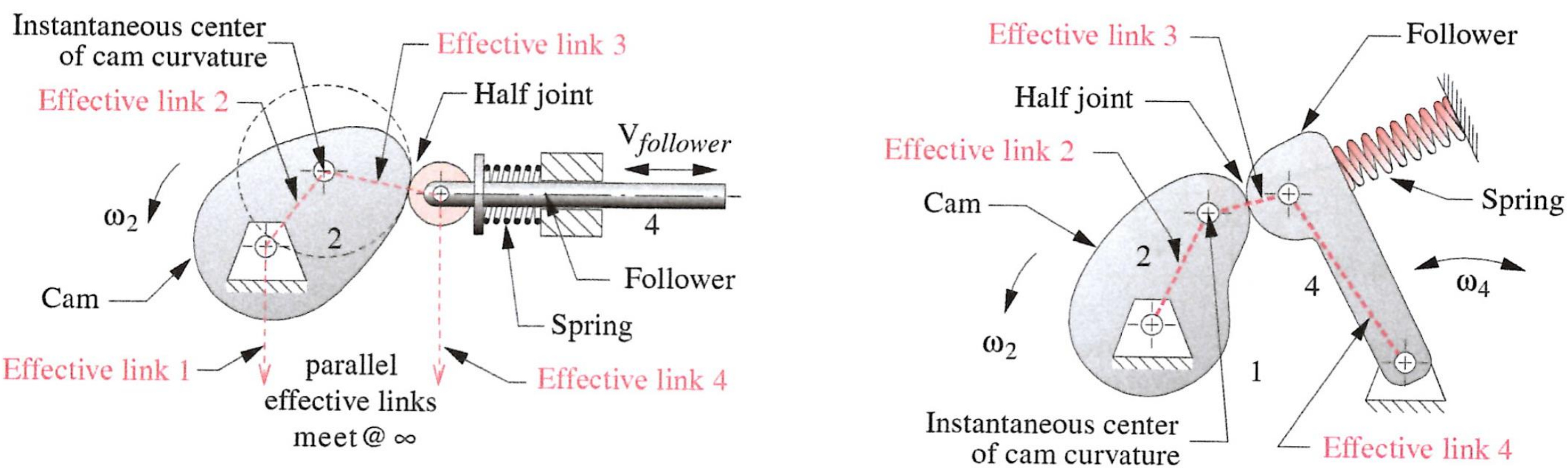
(a) Crank-slider—crank drives slider



(b) Slider-crank—slider drives crank

Cam and Follower

- *The crank takes a contoured shape.*
- *Can be viewed as a fourbar crank-slider or slider-crank in which the crank and coupler are able to change their lengths as the cam rotates.*



(*a*) Cam with sliding follower—a variant of a crank-slider

(*b*) Cam with oscillating follower—a variant of a crank-rocker

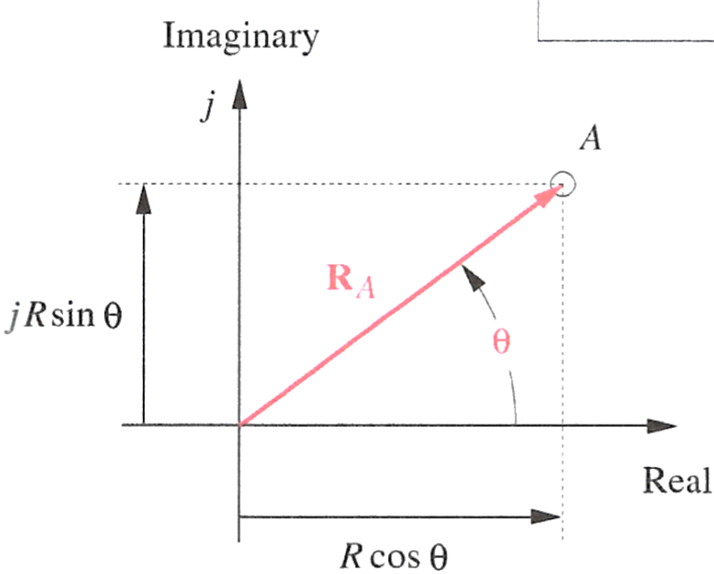
Analyzing Linkage Motion

$$e^{\pm j\theta} = \cos\theta \pm j\sin\theta$$

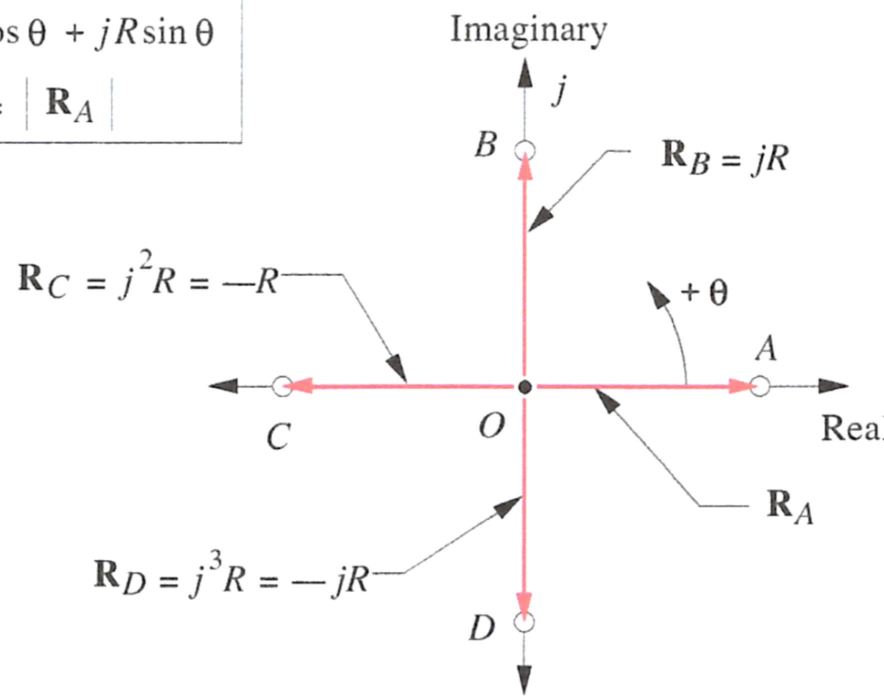
Polar form: $R e^{j\theta}$

Cartesian form: $R\cos\theta + jR\sin\theta$

$R = |\mathbf{R}_A|$



(a) Complex number representation of a position vector



(b) Vector rotations in the complex plane

Analyzing the Fourbar Linkage

Vector Loop Equation $\mathbf{R}_2 + \mathbf{R}_3 - \mathbf{R}_4 - \mathbf{R}_1 = 0$ → $a e^{j\theta_2} + b e^{j\theta_3} - c e^{j\theta_4} - d e^{j\theta_1} = 0$

$$a(\cos\theta_2 + j\sin\theta_2) + b(\cos\theta_3 + j\sin\theta_3) - c(\cos\theta_4 + j\sin\theta_4) - d(\cos\theta_1 + j\sin\theta_1) = 0$$

Real Part

$$a\cos\theta_2 + b\cos\theta_3 - c\cos\theta_4 - d\cos\theta_1 = 0$$

but: $\theta_1 = 0$, so:

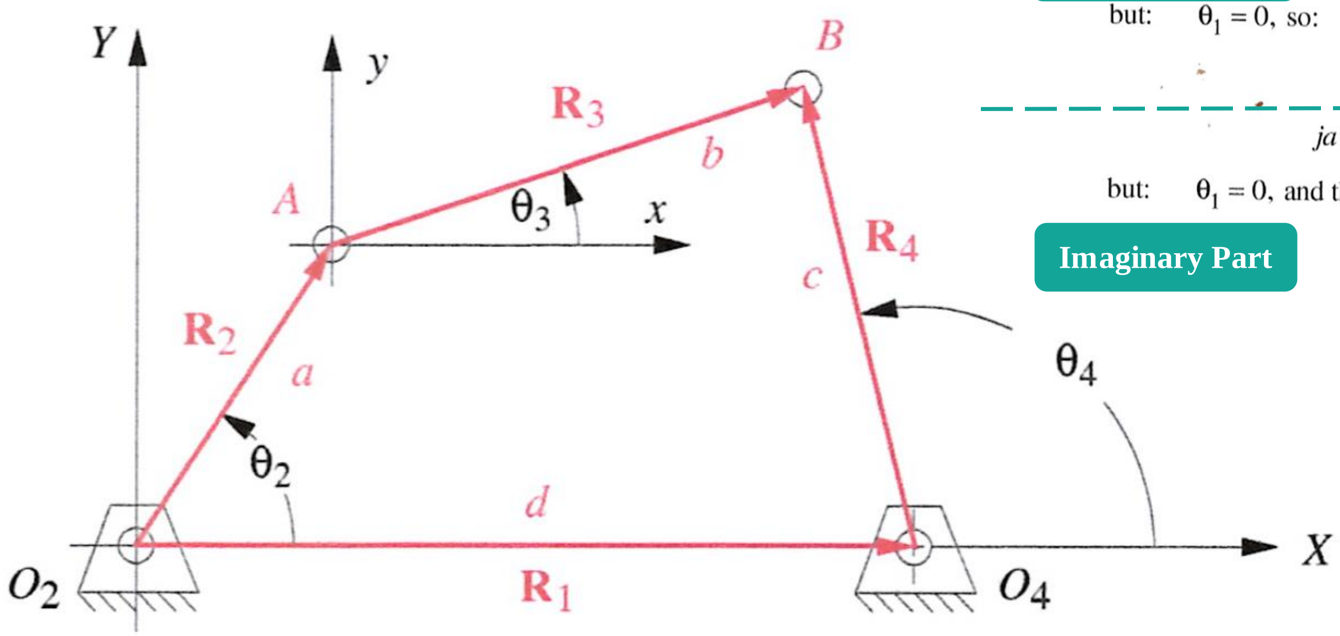
$$a\cos\theta_2 + b\cos\theta_3 - c\cos\theta_4 - d = 0$$

$$ja\sin\theta_2 + jb\sin\theta_3 - jc\sin\theta_4 - jd\sin\theta_1 = 0$$

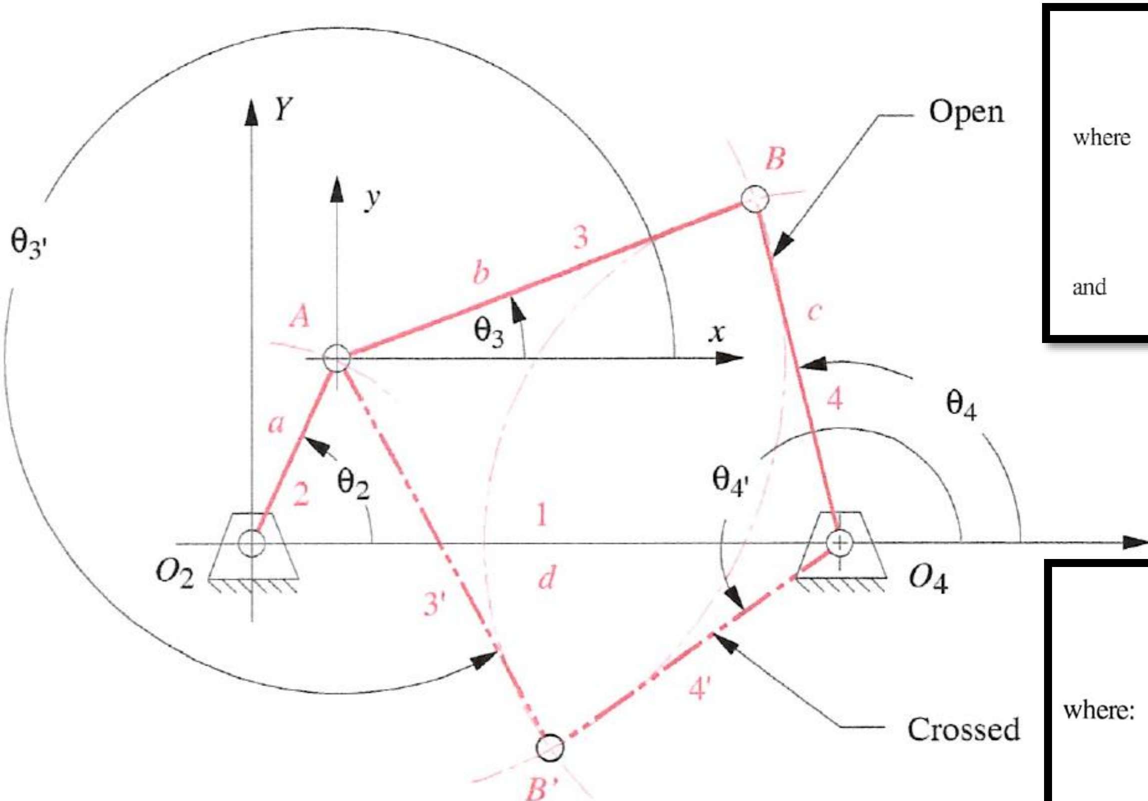
but: $\theta_1 = 0$, and the j's divide out, so:

Imaginary Part

$$a\sin\theta_2 + b\sin\theta_3 - c\sin\theta_4 = 0$$



The Two Solutions to theta_3/4 of the Fourbar **Position** Equation



$$\theta_{3_{1,2}} = 2\arctan\left(\frac{-E \pm \sqrt{E^2 - 4DF}}{2D}\right)$$

where

$$D = \cos\theta_2 - K_1 + K_4\cos\theta_2 + K_5$$
$$E = -2\sin\theta_2$$
$$F = K_1 + (K_4 - 1)\cos\theta_2 + K_5$$

and

$$K_1 = \frac{d}{a} \qquad K_4 = \frac{d}{b} \qquad K_5 = \frac{c^2 - d^2 - a^2 - b^2}{2ab}$$

$$\theta_{4_{1,2}} = 2\arctan\left(\frac{-B \pm \sqrt{B^2 - 4AC}}{2A}\right)$$

where:

$$A = \cos\theta_2 - K_1 - K_2\cos\theta_2 + K_3$$
$$B = -2\sin\theta_2$$
$$C = K_1 - (K_2 + 1)\cos\theta_2 + K_3$$

and:

$$K_1 = \frac{d}{a}, \qquad K_2 = \frac{d}{c}, \qquad K_3 = \frac{a^2 - b^2 + c^2 + d^2}{2ac}$$

Velocity Equation of the Fourbar Linkage

Vector Loop Equation

$$\mathbf{R}_2 + \mathbf{R}_3 - \mathbf{R}_4 - \mathbf{R}_1 = 0$$

$$a e^{j\theta_2} + b e^{j\theta_3} - c e^{j\theta_4} - d e^{j\theta_1} = 0$$

Differentiate with respect to time to get velocity.

$$ja e^{j\theta_2}\frac{d\theta_2}{dt} + jb e^{j\theta_3}\frac{d\theta_3}{dt} - jc e^{j\theta_4}\frac{d\theta_4}{dt} = 0$$

but, $\dfrac{d\theta_2}{dt} = \omega_2;\quad \dfrac{d\theta_3}{dt} = \omega_3;\quad \dfrac{d\theta_4}{dt} = \omega_4$

so: $$ja\omega_2 e^{j\theta_2} + jb\omega_3 e^{j\theta_3} - jc\omega_4 e^{j\theta_4} = 0$$

$$\mathbf{V}_A = ja\omega_2(\cos\theta_2 + j\sin\theta_2) = a\omega_2(-\sin\theta_2 + j\cos\theta_2)$$

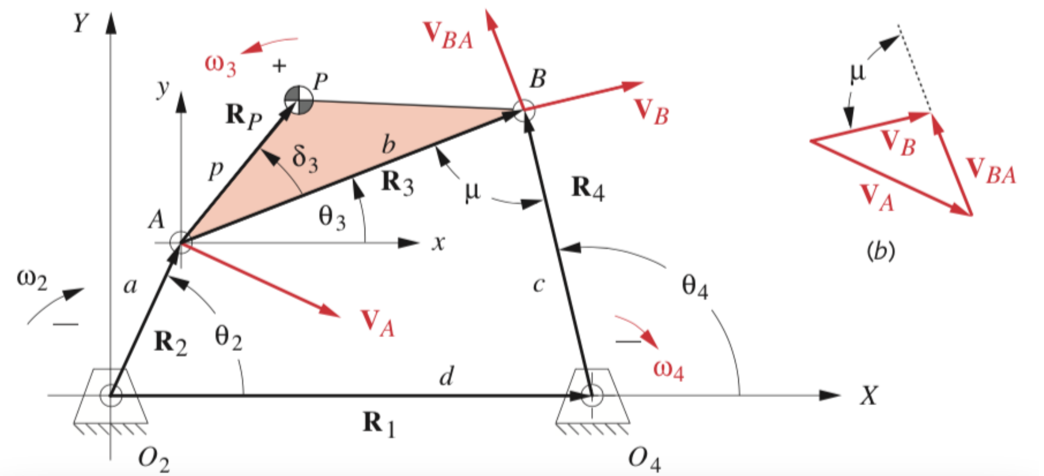
$$\mathbf{V}_{BA} = jb\omega_3(\cos\theta_3 + j\sin\theta_3) = b\omega_3(-\sin\theta_3 + j\cos\theta_3)$$

$$\mathbf{V}_B = jc\omega_4(\cos\theta_4 + j\sin\theta_4) = c\omega_4(-\sin\theta_4 + j\cos\theta_4)$$

$$e^{\pm j\theta} = \cos\theta \pm j\sin\theta$$

$$\omega_3 = \frac{a\omega_2}{b}\frac{\sin(\theta_4 - \theta_2)}{\sin(\theta_3 - \theta_4)}$$

$$\omega_4 = \frac{a\omega_2}{c}\frac{\sin(\theta_2 - \theta_3)}{\sin(\theta_4 - \theta_3)}$$



Acceleration Equation of the Fourbar Linkage

Vector Loop Equation

$$\mathbf{R}_2 + \mathbf{R}_3 - \mathbf{R}_4 - \mathbf{R}_1 = 0 \quad \Rightarrow \quad a e^{j\theta_2} + b e^{j\theta_3} - c e^{j\theta_4} - d e^{j\theta_1} = 0$$

Differentiate with respect to time to get ***velocity****.*

$$ja\omega_2 e^{j\theta_2} + jb\omega_3 e^{j\theta_3} - jc\omega_4 e^{j\theta_4} = 0$$

Differentiate with respect to time to get ***acceleration****.*

$$\left(j^2 a\omega_2^2 e^{j\theta_2} + ja\alpha_2 e^{j\theta_2}\right) + \left(j^2 b\omega_3^2 e^{j\theta_3} + jb\alpha_3 e^{j\theta_3}\right) - \left(j^2 c\omega_4^2 e^{j\theta_4} + jc\alpha_4 e^{j\theta_4}\right) = 0$$

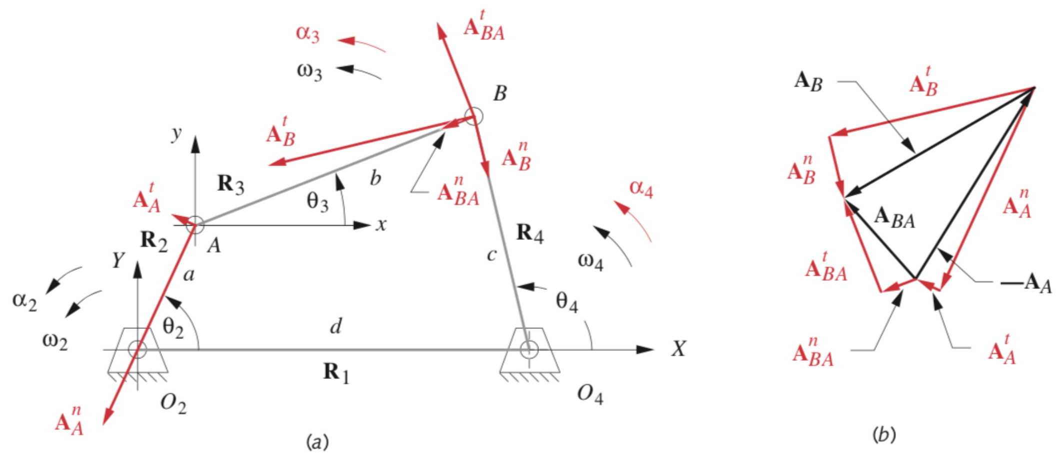
Rewrite for physical meaning.

$$\mathbf{A}_A + \mathbf{A}_{BA} - \mathbf{A}_B = 0$$

$$\mathbf{A}_A = \left(\mathbf{A}_A^t + \mathbf{A}_A^n\right) = \left(a\alpha_2 je^{j\theta_2} - a\omega_2^2 e^{j\theta_2}\right)$$

$$\mathbf{A}_{BA} = \left(\mathbf{A}_{BA}^t + \mathbf{A}_{BA}^n\right) = \left(b\alpha_3 je^{j\theta_3} - b\omega_3^2 e^{j\theta_3}\right)$$

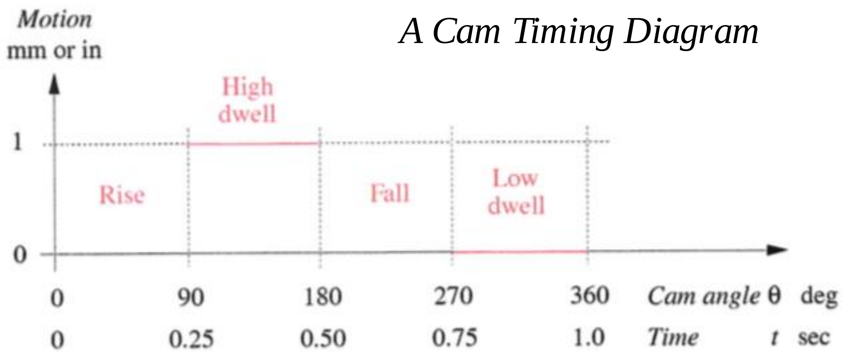
$$\mathbf{A}_B = \left(\mathbf{A}_B^t + \mathbf{A}_B^n\right) = \left(c\alpha_4 je^{j\theta_4} - c\omega_4^2 e^{j\theta_4}\right)$$



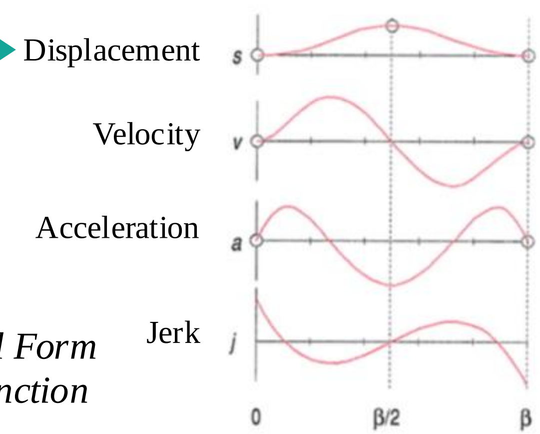
Cam Design & Analysis

- Dwell
  - The cessation of follower motion while the cam rotation continues for a portion of the cycle

A Cam Timing Diagram



svaj Diagram



$$s = C_0 + C_1 x + C_2 x^2 + C_3 x^3 + C_4 x^4 + C_5 x^5 + C_6 x^6 + \cdots + C_n x^n$$

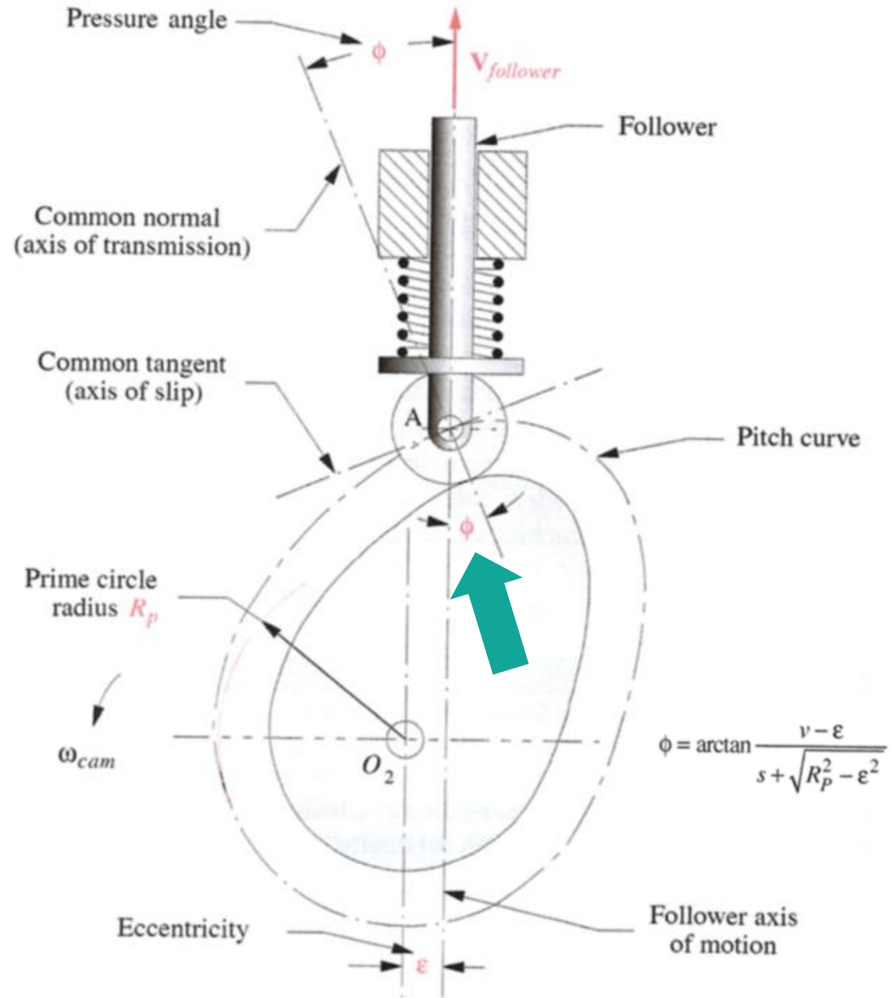
The crank takes a contoured shape.

General Polynomial Form of Displacement Function

- Fundamental Law of Cam Design
  - *The motion function must be piecewise continuous over the entire cam through the second derivative of displacement.*
  - ***No discontinuities in the position, velocity, or acceleration functions over the full cycle of the cam.***

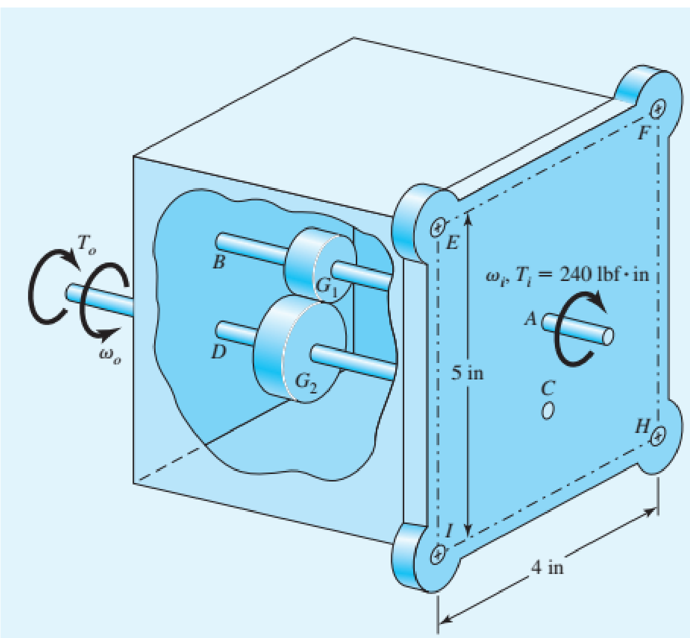
Pressure Angle

- Definition
  - The angle between the velocity of the follower and the common normal at the contact point between cam and follower.
- Rule of thumb
  - Keep the maximum pressure angle below about 30 degree for a translating follower.



Equilibrium & Free-body Diagrams

- **Equilibrium**: *A System with Zero Acceleration.*
- **Free-body Diagrams**: *A means of breaking a complicated problem into manageable segments, analyzing these simple problems, and then, usually, putting the information together again.*



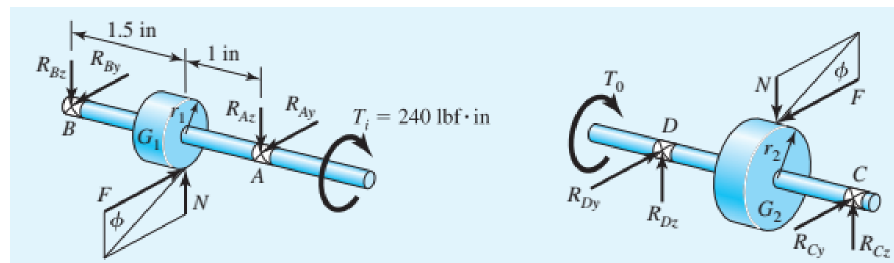
$$\sum \mathbf{F} = 0$$

$$\sum \mathbf{M} = 0$$

$$\sum M_x = F(0.75) - 240 = 0$$

$$F = 320 \text{ lbf}$$

R_{Ay} = 192 lbf, R_{Az} = 69.9 lbf,
R_{By} = 128 lbf, R_{Bz} = 46.6 lbf,
R_{Cy} = 192 lbf, R_{Cz} = 69.9 lbf,
R_{Dy} = 128 lbf, R_{Dz} = 46.6 lbf,
T_o = 480 lbf · in.



(*c*) Input shaft

(*d*) Output shaft

Load Analysis

- Newton's **First** Law *(Governing Assumption)*
  - *A body at rest tends to remain at rest and a body in motion at constant velocity will tend to maintain that velocity unless acted upon by an external force.*
- Newton's **Second** Law *(Calculated Guess)*
  - *The time rate of change of momentum of a body is equal to the magnitude of the applied force and acts in the direction of the force.*
- Newton's **Third** Law *(Interactive Analysis)*
  - *When two particles interact, a pair of equal and opposite reaction forces will exist at their contact point. This force pair will have the same magnitude and act along the same direction line, but have opposite sense.*

$$\sum \mathbf{F} = m\mathbf{a} \qquad \sum F_x = ma_x \qquad \sum F_y = ma_y \qquad \sum F_z = ma_z$$

$$\sum \mathbf{M}_G = \dot{\mathbf{H}}_G \qquad \mathbf{H}_G = I_x\omega_x\hat{\mathbf{i}} + I_y\omega_y\hat{\mathbf{j}} + I_z\omega_z\hat{\mathbf{k}}$$

I_x, I_y, and I_z are the principal centroidal mass moments of inertia

These 6 equations can be written for each rigid body in a 3-D system

$$\sum M_x = I_x\alpha_x - (I_y - I_z)\omega_y\omega_z$$
$$\sum M_y = I_y\alpha_y - (I_z - I_x)\omega_z\omega_x$$
$$\sum M_z = I_z\alpha_z - (I_x - I_y)\omega_x\omega_y$$

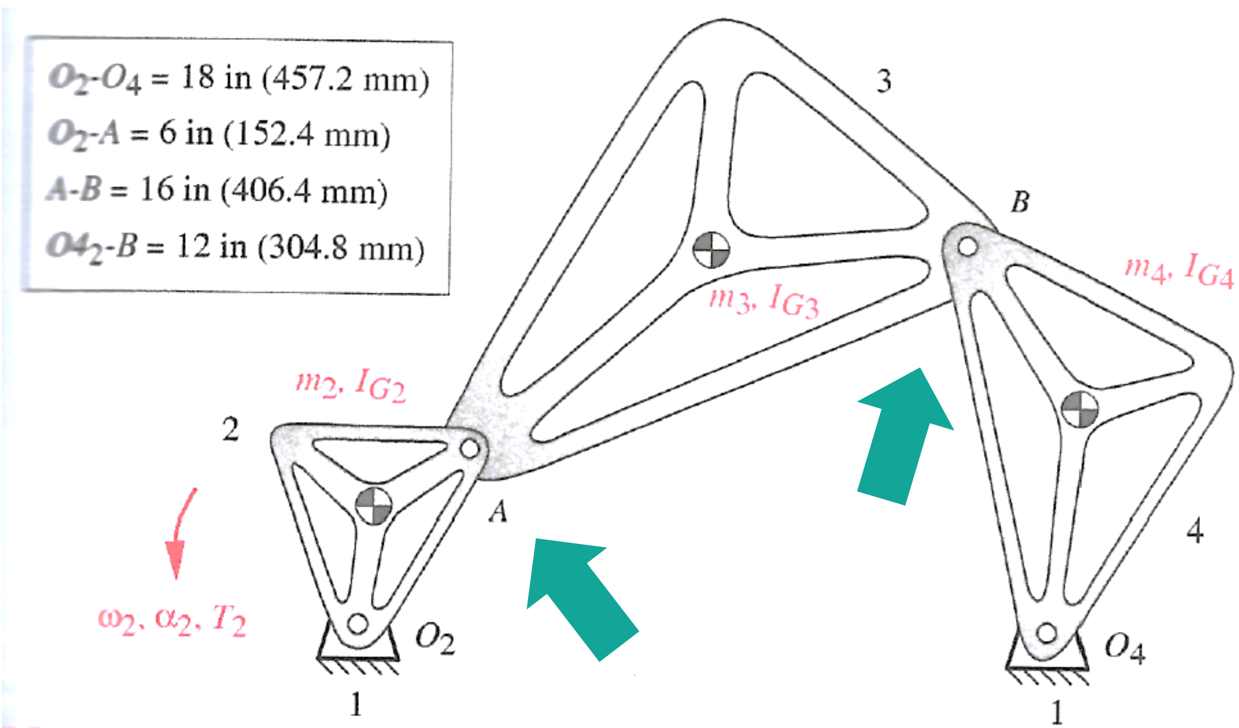
Moments *Angular Accelerations*

Needs to be solved simultaneously for the forces and moments

Total # of Load Eqs. to solve for a system = *# of rigid bodies the system* × *(3 Force Eqs. + 3 Moment Eqs.)* + *Reaction Eqs.*

Fourbar Linkage Loading Analysis

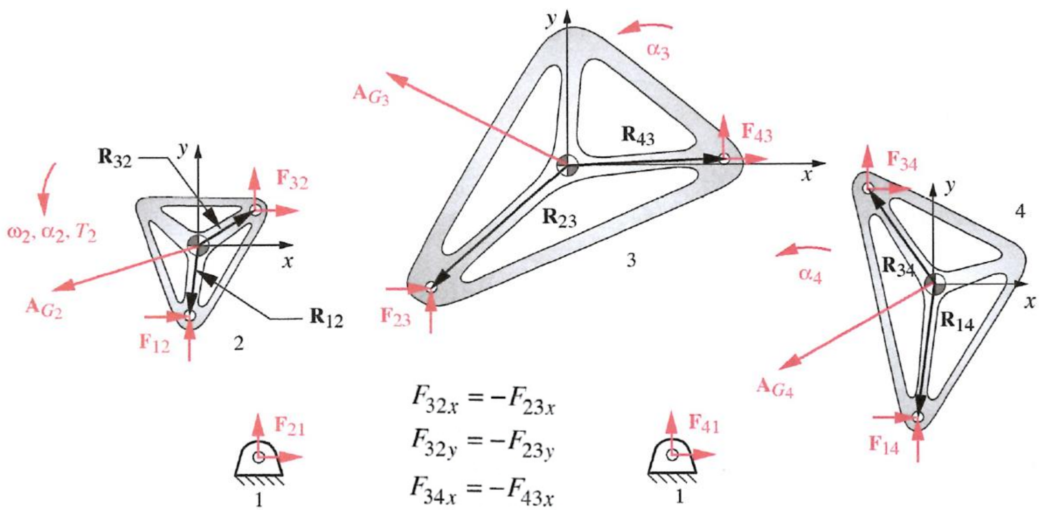
- [**Problem**] Determine the theoretical rigid body **forces** acting in two dimensions on the fourbar linkage.
- [**Given**] The linkage geometry, masses, and mass moments of inertia are known and the linkage is driven at up to 120 rpm by a speed-controlled electric motor.



Free-Body Diagrams

Given and Assumed Data

Variable	Value	Unit
θ_2	30.00	deg
ω_2	120.00	rpm
$mass_2$	0.525	kg
$mass_3$	1.050	kg
$mass_4$	1.050	kg
I_{cg2}	0.057	kg-m^2
I_{cg3}	0.011	kg-m^2
I_{cg4}	0.455	kg-m^2
R_{12x}	−46.9	mm
R_{12y}	−71.3	mm
R_{32x}	85.1	mm
R_{32y}	4.9	mm
R_{23x}	−150.7	mm
R_{23y}	−177.6	mm
R_{43x}	185.5	mm
R_{43y}	50.8	mm
R_{14x}	−21.5	mm
R_{14y}	−100.6	mm
R_{34x}	−10.6	mm
R_{34y}	204.0	mm



$$F_{32x} = -F_{23x}$$
$$F_{32y} = -F_{23y}$$
$$F_{34x} = -F_{43x}$$
$$F_{34y} = -F_{43y}$$

For link 2:

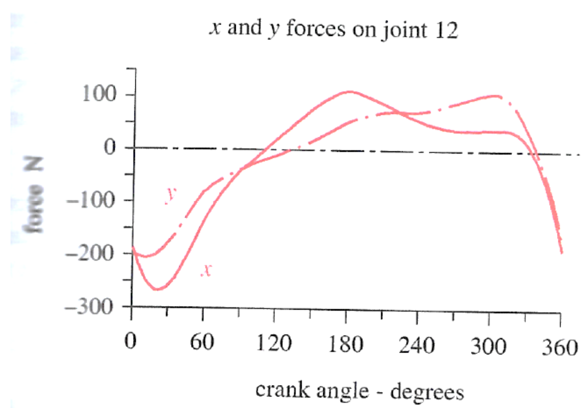
$$\sum F_x = F_{12x} + F_{32x} = m_2 a_{G2_x}$$
$$\sum F_y = F_{12y} + F_{32y} = m_2 a_{G2_y}$$
$$\sum M_z = T_2 + \left(R_{12x}F_{12y} - R_{12y}F_{12x}\right) + \left(R_{32x}F_{32y} - R_{32y}F_{32x}\right) = I_{G2}\alpha_2$$

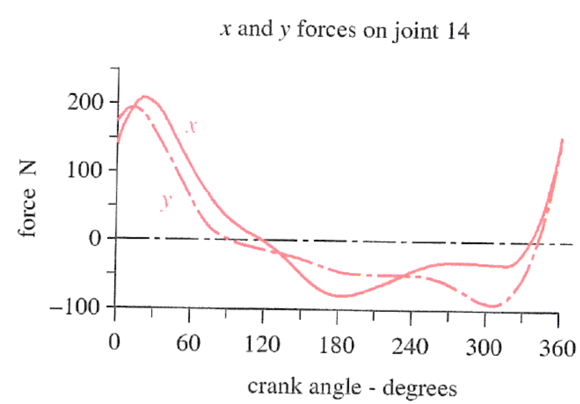
For link 3:

$$\sum F_x = F_{23x} + F_{43x} = m_3 a_{G3_x}$$
$$\sum F_y = F_{23y} + F_{43y} = m_3 a_{G3_y}$$
$$\sum M_z = \left(R_{23x}F_{23y} - R_{23y}F_{23x}\right) + \left(R_{43x}F_{43y} - R_{43y}F_{43x}\right) = I_{G3}\alpha_3$$

For link 4:

$$\sum F_x = F_{14x} + F_{34x} = m_4 a_{G4_x}$$
$$\sum F_y = F_{14y} + F_{34y} = m_4 a_{G4_y}$$
$$\sum M_z = \left(R_{14x}F_{14y} - R_{14y}F_{14x}\right) + \left(R_{34x}F_{34y} - R_{34y}F_{34x}\right) = I_{G4}\alpha_4$$





Vibration Loading

- All real elements of any material have elasticity
  - Thus act as spring when subjected to forces
    - Causing deflection to produce additional forces to be generated from the inertial forces associated with the vibratory movement of elements;
    - If clearances allow contact of mating parts, may generate impact (shock) loads during their vibrations.

- How to predict?
  - Modern finite element (FEA) or boundary element (BEA) analysis techniques are good ways to model and calculate
    - Break up the assembly into a large number of discrete elements
    - Limited by time and the computing resources available
  - Field or Scaled experimentations

- How to eliminate?
  - Better Design or Better Design Engineers

Natural Frequency & Dynamic Forces

- A system's lowest frequency (*a calculated estimation*)
  - Usually creates the largest magnitude of vibration

undamped fundamental natural frequency ω_n: $\omega_n = \sqrt{\frac{k}{m}}$

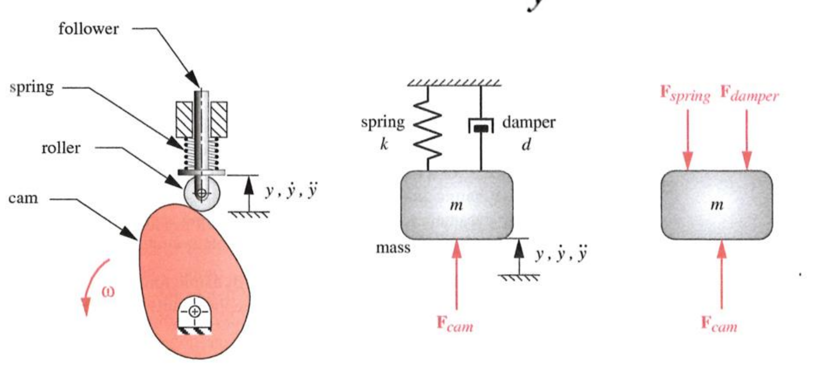
$$f_n = \frac{1}{2\pi}\omega_n$$

$$\sum F_y = ma = m\ddot{y}$$

$$F_{cam} - F_{spring} - F_{damper} = m\ddot{y}$$

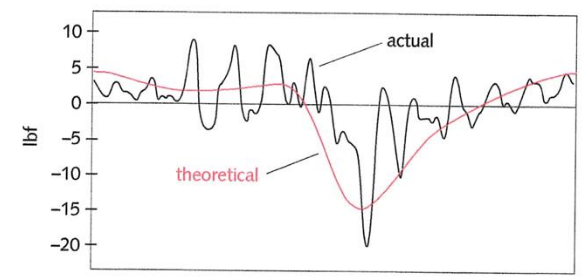
$$F_{cam} = m\ddot{y} + d\dot{y} + ky$$

- Metrics
  - Spring Constant k $\quad k = \frac{F}{y}$
  - Damping d $\quad d = \frac{F}{\dot{y}}$

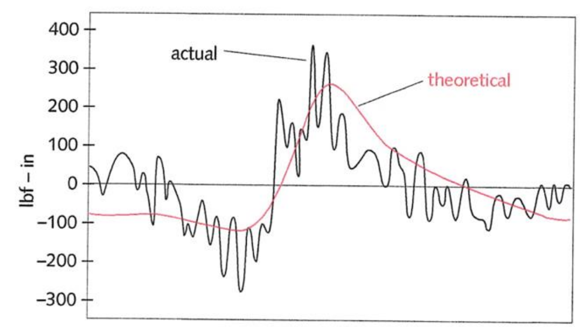


(a) Actual system (b) Lumped model (c) Free-body diagram

(a) Theoretical and actual dynamic force in x direction at crank pivot



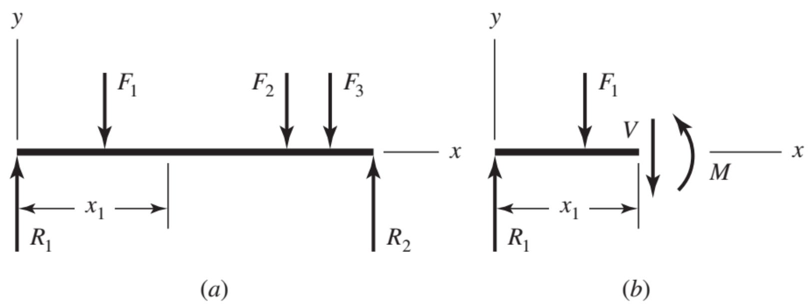
(b) Theoretical and actual dynamic torque at crank pivot



Beam Loading

Figure 3–2

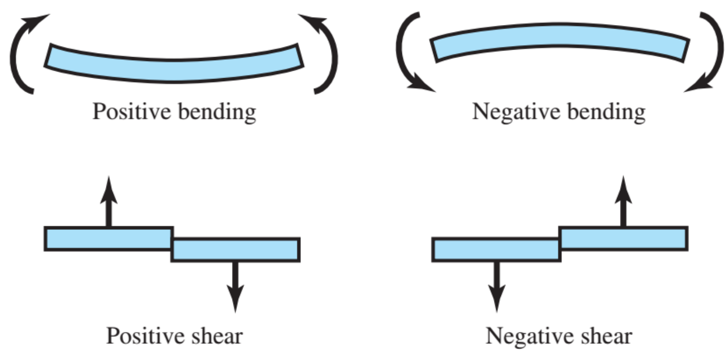
Free-body diagram of simply-supported beam with V and M shown in positive directions (established by the conventions shown in Fig. 3–3).



If the beam is cut at some section located at $x = x\_1$ and the left-hand portion is removed as a free body, an internal shear force V and bending moment M must act on the cut surface to ensure equilibrium.

Figure 3–3

Sign conventions for bending and shear.



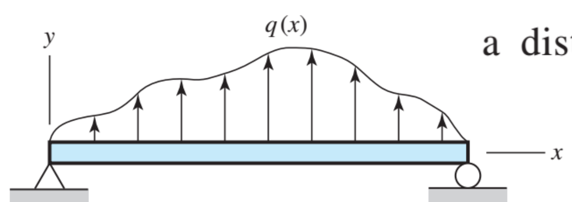
$$V = \frac{dM}{dx}$$

$$\int_{V_A}^{V_B} dV = V_B - V_A = \int_{x_A}^{x_B} q\, dx$$

$$\int_{M_A}^{M_B} dM = M_B - M_A = \int_{x_A}^{x_B} V\, dx$$

Figure 3–4

Distributed load on beam.

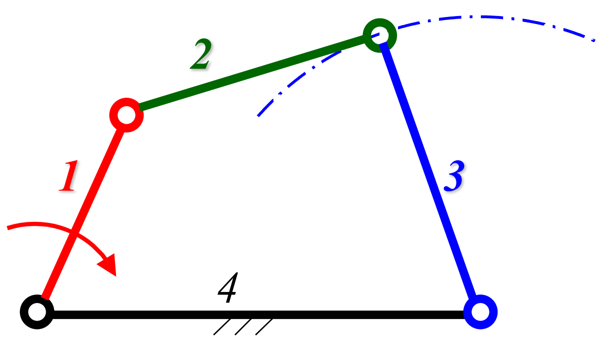


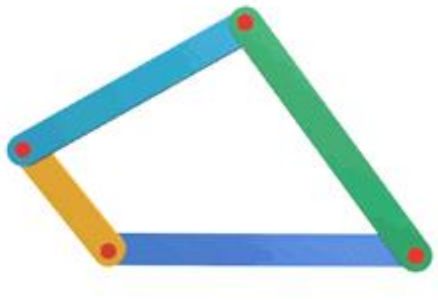
a distributed load $q(x)$

$$\frac{dV}{dx} = \frac{d^2M}{dx^2} = q$$

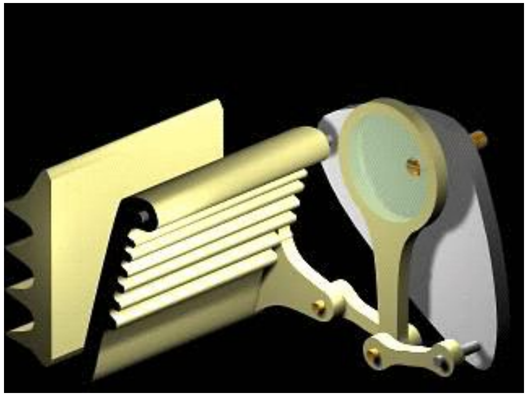
Advanced Textbooks are recommended to further study this topic in depth.

平面四杆机构的基本形式、演变及其应用





曲柄摇杆机构



- 在平面四杆机构的两连架杆中
  - 若一个为曲柄，而另一个为摇杆

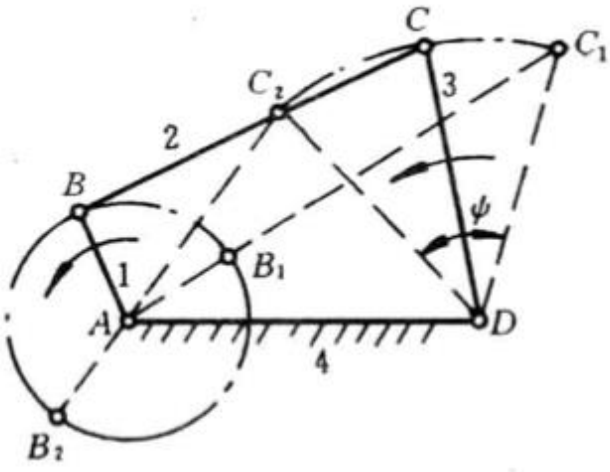
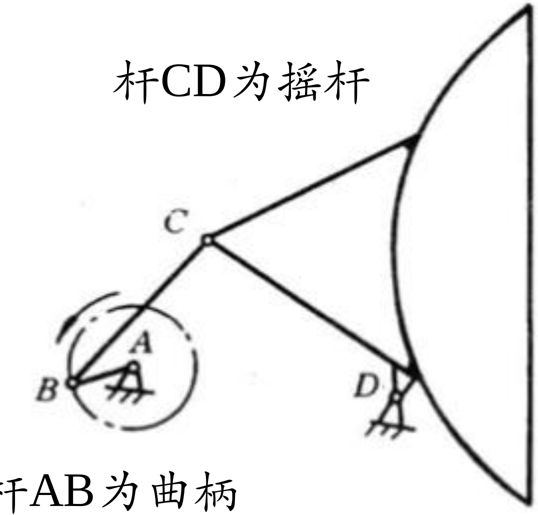


图 2-1 曲柄摇杆机构

杆CD为摇杆



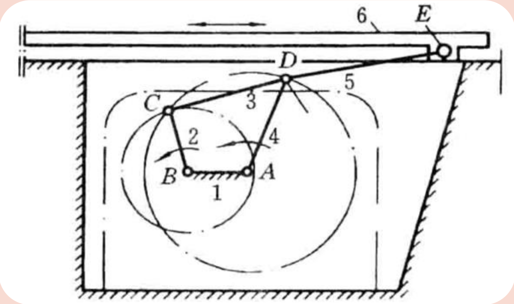
杆AB为曲柄

图 2-2 雷达天线机构

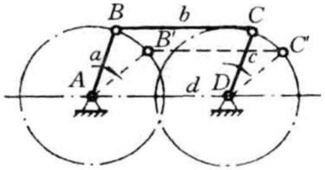
利用了曲柄摇杆机构调节天线的俯仰角

双曲柄机构

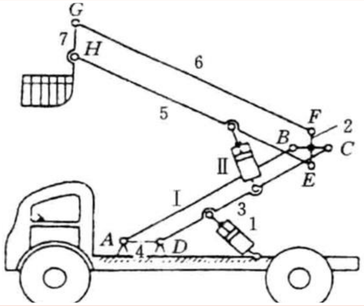
- 若平面四杆机构的两连架杆均为曲柄



惯性筛双曲柄机构



平行四边形机构



摄影车升降机构

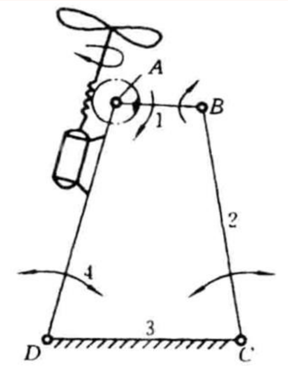
当曲柄 2 等速回转时，另一曲柄 4 作变速回转，使筛子具有所需的加速度，再利用加速度所产生的惯性力，使大小不同的颗粒在筛上作往复运动，从而达到筛选的目的。

在双曲柄机构中，若两组对边的构件长度相等，则可得到平行四边形机构，由于这种机构两连架杆的运动完全相同，故连杆始终作平动。

利用平行四边形机构的连杆始终作平动的特点，使与连杆固结在一起的座椅始终保持水平位置，其升降高度的变化也是通过采用两套平行四边形机构来实现的。

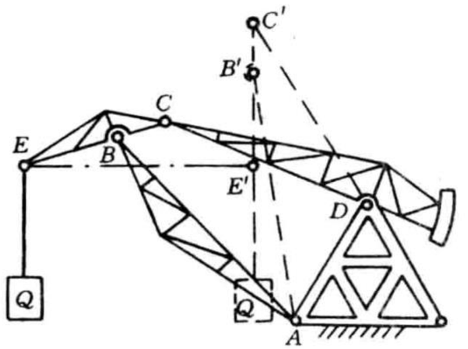
双摇杆机构

- 在平面四杆机构的两连架杆中
  - 若两连架杆**均为摇杆**



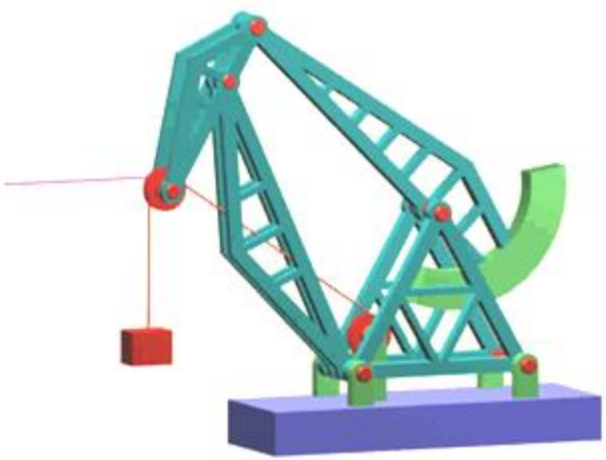
摇头风扇传动机构

电动机安装在摇杆4上，铰链A处装有一个与连杆1固连成一体的蜗轮，并与电动机轴上的蜗杆相啮合；电动机转动时，通过蜗杆和蜗轮迫使连杆1绕点A作整周转动，从而使连架杆2和4往复摆动，实现风扇摇头的目的



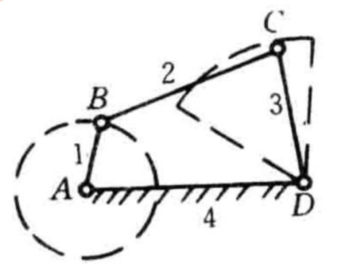
鹤式起重机

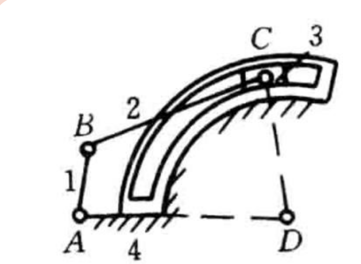
当摇杆AB摆动时，另一摇杆CD随之摆动，可使吊在连杆上E处的重物Q能沿近似水平直线移动

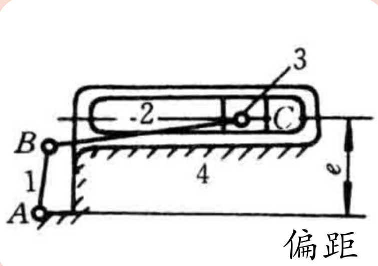


平面四杆机构的演变

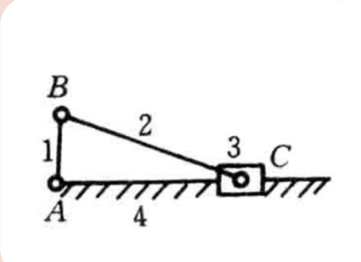
- 转动副**转化成**移动副
  - 含滑块的平面四杆机构均可看成由铰链四杆机构演变而成







偏置滑块机构



对心滑块机构

摇杆3上点C的运动轨迹是以点D为圆心，以摇杆长度 l_{CD} 为半径所作的圆弧

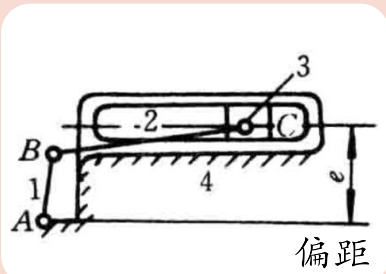
若改为如图所示的形式，则机构运动的特性完全一样

若此弧形槽的半径增至无穷大（即点D在无穷远处），则弧形槽变成直槽，**转动副也就转化成移动副**，此时构件3也就由摇杆变成了滑块

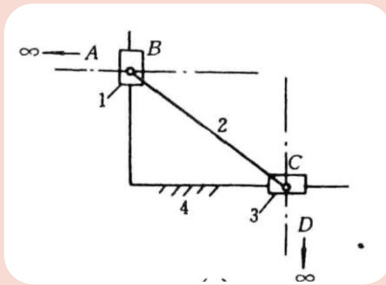
若滑块上的转动副中心的移动方位线通过曲柄的回转中心，称这种滑块机构为**对心滑块机构**

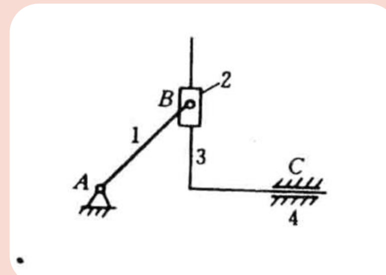
平面四杆机构的演变

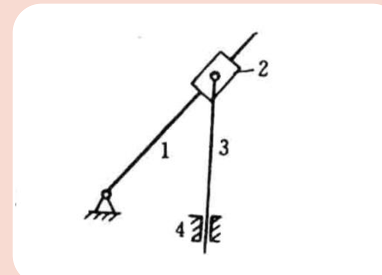
- 含有**两个移动副**的四杆机构



偏置滑块机构







若此弧形槽的半径增至无穷大（即点D在无穷远处），则弧形槽变成直槽，**转动副也就转化成移动副**，此时构件3也就由摇杆变成了滑块

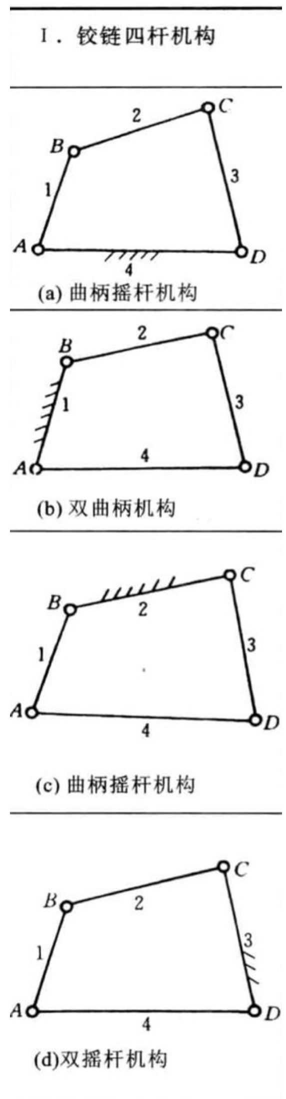
如将点A移至无穷远处，则转动副A演变成移动副，得到**双滑块机构**

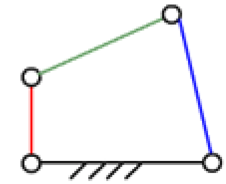
也可将构件2与构件3之间的转动副C变成移动副，得到**曲柄移动导杆机构（又称正弦机构）**

若将转动副B变成移动副，则可得到**正切机构**

取不同构件为机架

- 机构的倒置
  - 低副机构具有运动可逆性
  - **<u>无论哪一个构件为机架，机构中各构件间的相对运动不变</u>**，但选取不同构件为机架时，可得到不同形式的机构

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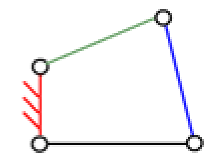
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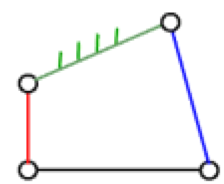
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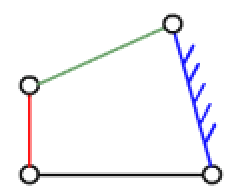
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曲柄摇杆机构

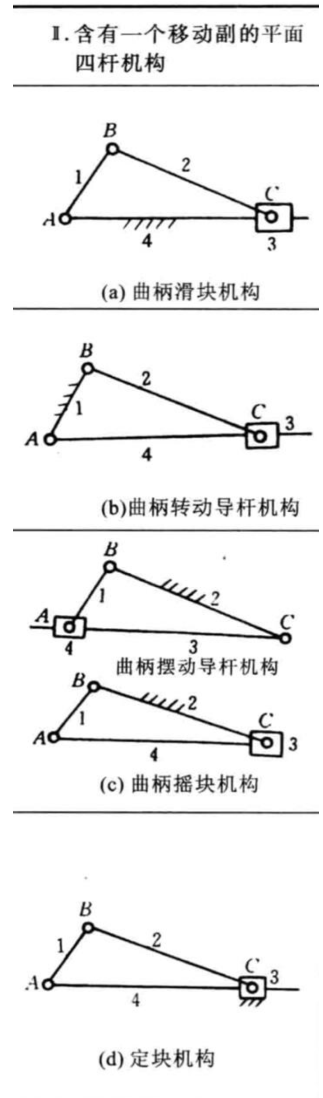
- 双曲柄机构
- 曲柄摇杆机构
- 双摇杆机构

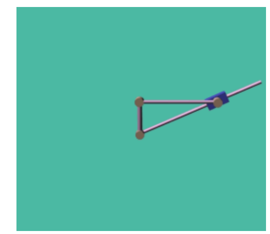
取不同构件为机架

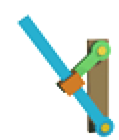
- 机构的倒置
  - 低副机构具有运动可逆性
  - **无论哪一个构件为机架，机构中各构件间的相对运动不变**，但选取不同构件为机架时，可得到不同形式的机构

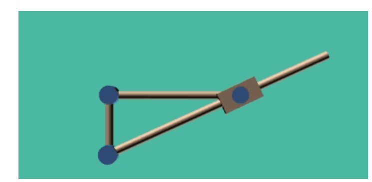
曲柄滑块机构

- 曲柄转动导杆机构
- 曲柄摇块机构
- 定块机构







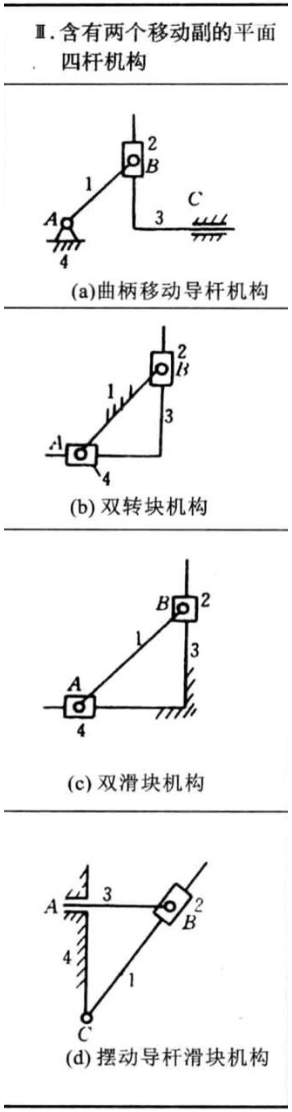


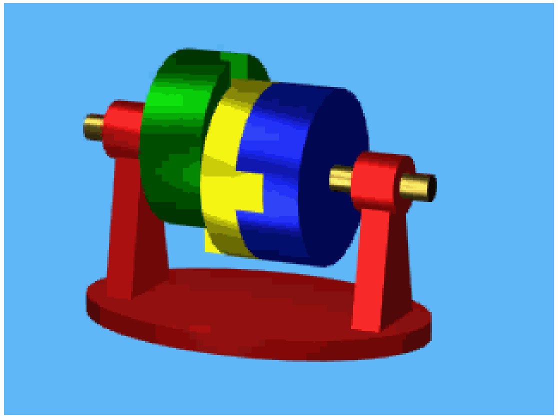
取不同构件为机架

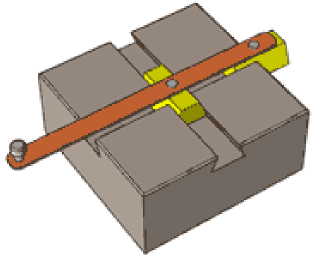
- 机构的倒置
  - 低副机构具有运动可逆性
  - **无论哪一个构件为机架，机构中各构件间的相对运动不变**，但选取不同构件为机架时，可得到不同形式的机构

曲柄移动导杆机构

- 双转块机构
- 双滑块机构
- 摆动导杆滑块机构



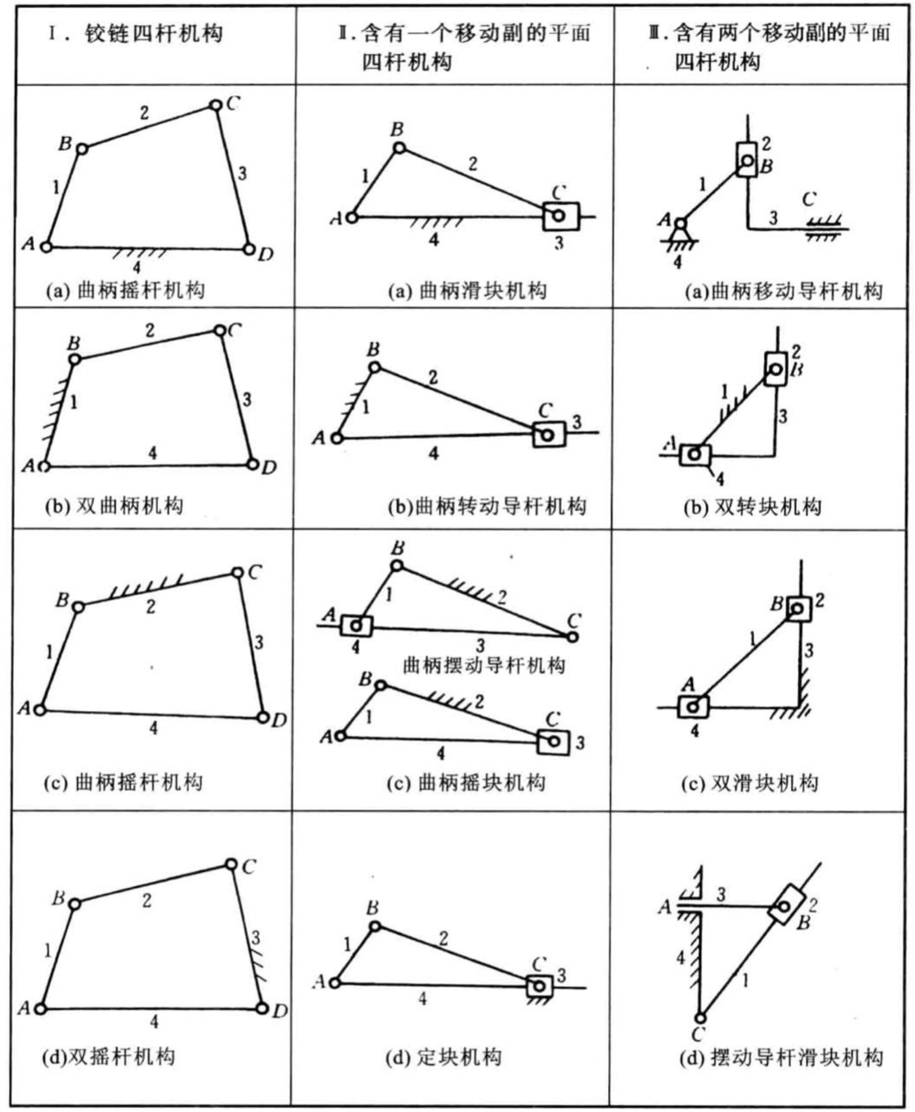




取不同构件为机架

- 机构的倒置
  - 低副机构具有运动可逆性
  - **无论哪一个构件为机架，机构中各构件间的相对运动不变**，但选取不同构件为机架时，可得到不同形式的机构

曲柄摇杆机构	曲柄滑块机构	曲柄移动导杆机构
双曲柄机构	曲柄转动导杆机构	双转块机构
曲柄摇杆机构	曲柄摇块机构	双滑块机构
双摇杆机构	定块机构	摆动导杆滑块机构



平面四杆机构设计中的共性问题

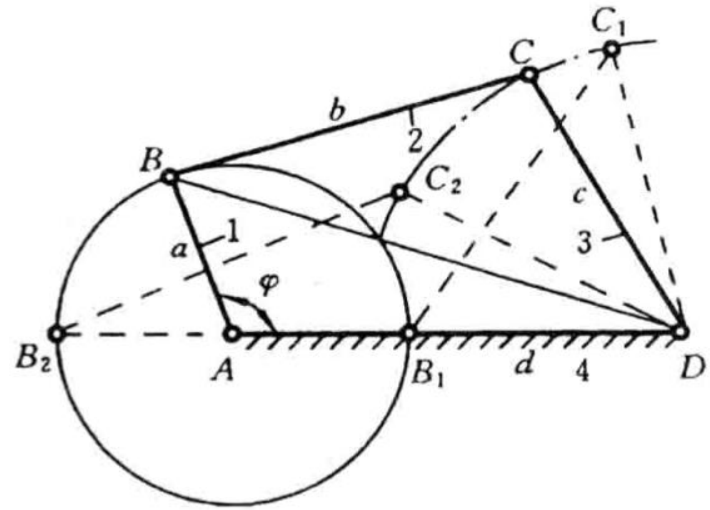
平面四杆机构存在曲柄的条件

在工程实际中，用于驱动机构运动的原动机通常是作整周转动的

要求机构的主动件也能作整周转动

即希望主动件是曲柄

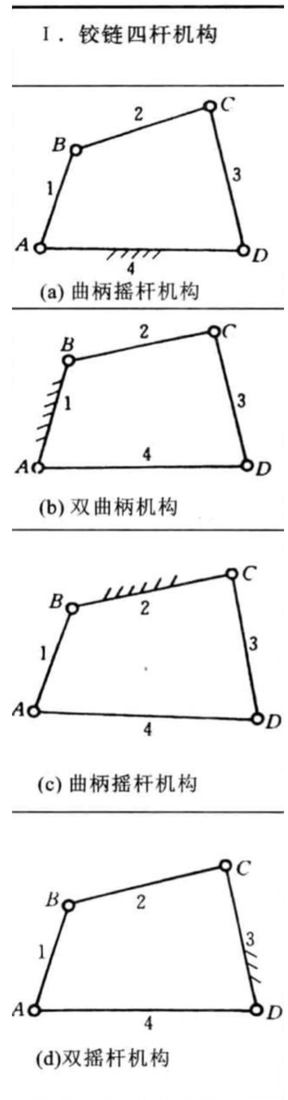
- 设平面四杆机构各杆 1、2、3 和 4 的长度分别为 a、b、c 和 d，杆 4 为机架，杆 1 和杆 3 为连架杆
- 当 $a < d$ 时，只要杆 1 能通过与机架两次共线的位置，则杆 1 必为曲柄
  - ΔB_2C_2D：$a + d \le b + c$
  - ΔB_1C_1D：$b \le (d - a) + c$ 或 $c \le (d - a) + b$
  - $a \le b$、$a \le c$、$a \le d$ => 即杆AB为最短杆
- 当 $d < a$ 时，同样分析可得
  - $d \le a$、$d \le b$、$d \le c$ => 即杆AD为最短杆

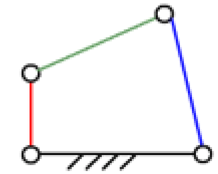


平面四杆机构存在曲柄的条件是:
(1) 连架杆与机架中必有一杆为平面四杆机构中的**最短杆**
(2) 最短杆与最长杆的杆长之和应小于或等于其余两杆的**杆长之和**

平面四杆机构存在曲柄的条件

- 在平面四杆机构中，如果最短杆与最长杆的长度之和小于或等于其他两杆长度之和，**且**：
  - ① **曲柄摇杆机构**： 以最短杆的相邻构件为机架，则最短杆为曲柄，另一连架杆为摇杆
  - ② **双曲柄机构**： 以最短杆为机架，则两连架杆均为曲柄
  - ③ **双摇杆机构**： 以最短杆的对边构件为机架，则无曲柄存在

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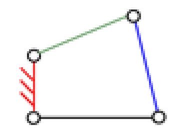
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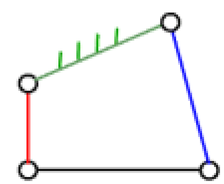
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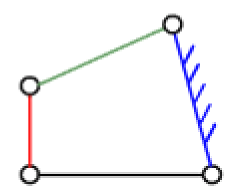
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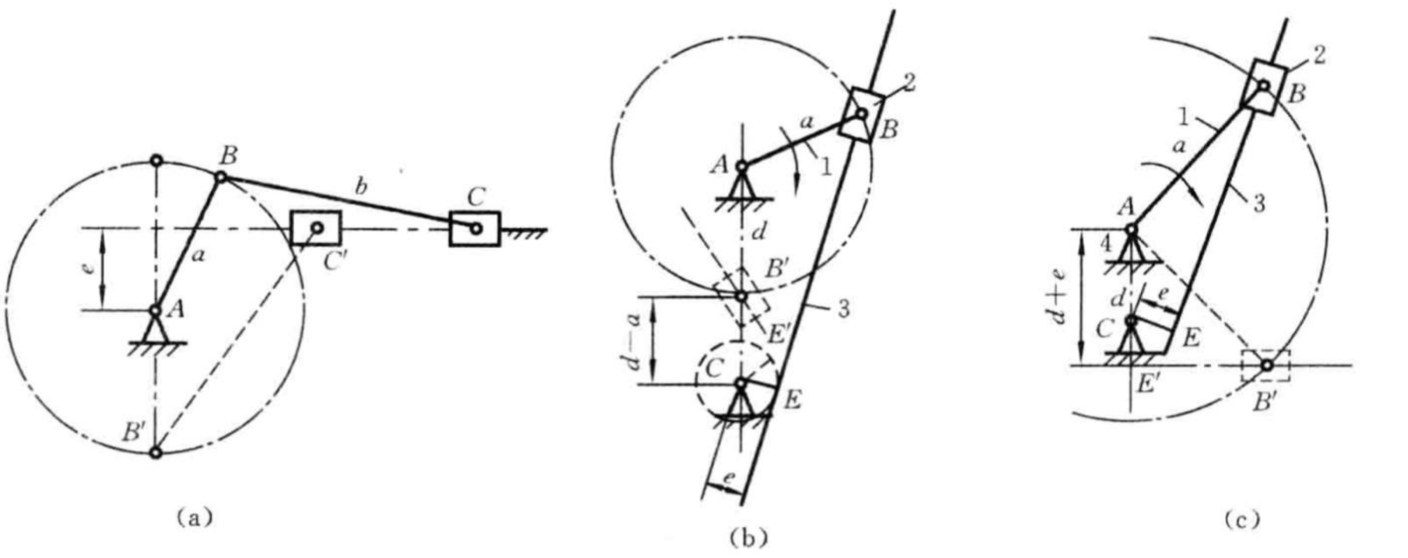
平面四杆机构存在曲柄的条件

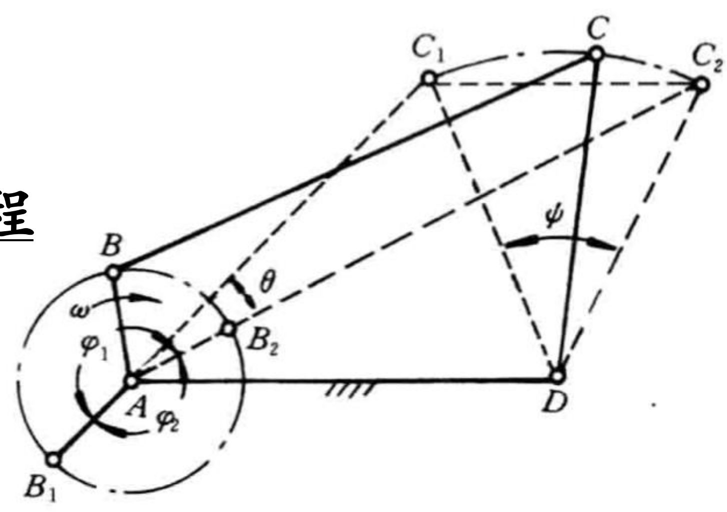
- 在平面四杆机构中，如果最短杆与最长杆的长度之和大于其他两杆长度之和，则不论选定哪一个构件为机架，均无曲柄存在，即该机构只能是双摇杆机构
  - 应当指出的是，在运用上述结论判断平面四杆机构的类型时，还应注意四个构件组成封闭多边形的条件，即**最长杆的杆长应小于其他三杆长度之和**
  - 对于图（a）中所示的**滑块机构**，可得到杆AB成为曲柄的条件是:
    - ① a为最短杆；② $a+e\le b$
  - 对于图（b）所示的**导杆机构**，可得到杆AB成为曲柄的条件是:
    - ① a为最短杆；② $a+e\le d$，这种机构称为曲柄摆动导杆机构
  - 对于图（c）所示的**曲柄转动导杆机构**，d为最短杆，且满足$d+e\le a$



平面四杆机构输出件的急回特性

- 当曲柄等速转动时，摇杆来回摆动的平均速度不同，一快一慢
  - 极位夹角 θ
    - $\varphi_1 = 180° + \theta > \varphi_2 = 180° - \theta \Rightarrow \ \varphi = \omega t \Rightarrow t_1 > t_2 \Rightarrow v_1 > v_2$
  - 有些机器（如刨床），要求从动件工作行程的速度低一些（以便提高加工质量），而为了提高机械的生产效率，要求返回行程的速度高一些

- **摇杆的急回特性**
  - 使机构的**慢速**运动的行程为**工作行程**
  - 而**快速**运动的行程为**空回行程**



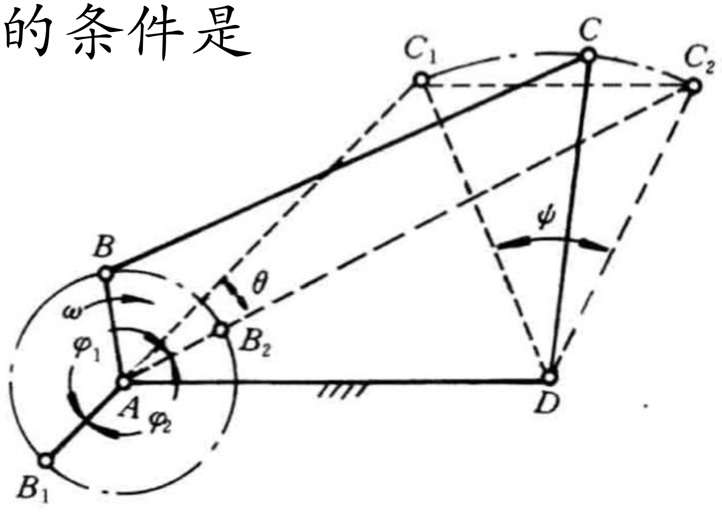
平面四杆机构输出件的急回特性

- 引入机构输出件的行程速度变化系数 k，表明急回运动的特征
  - k 的值为空回行程和工作行程的平均速度 v_2 与 v_1 的比值

$$\varphi_1 = 180° + \theta > \varphi_2 = 180° - \theta \Rightarrow \ \varphi = \omega t \Rightarrow t_1 > t_2 \Rightarrow v_1 > v_2$$

$$k = \frac{v_2}{v_1} = \frac{t_1}{t_2} = \frac{\varphi_1}{\varphi_2} = \frac{180° + \theta}{180° - \theta}$$

- 综上所述，平面四杆机构具有急回特性的条件是
  - (1) 原动件作等角速度整周转动
  - (2) 输出件作具有正、反行程的往复运动
  - (3) 极位夹角 $> 0^\circ$



平面四杆机构的传动角和死点

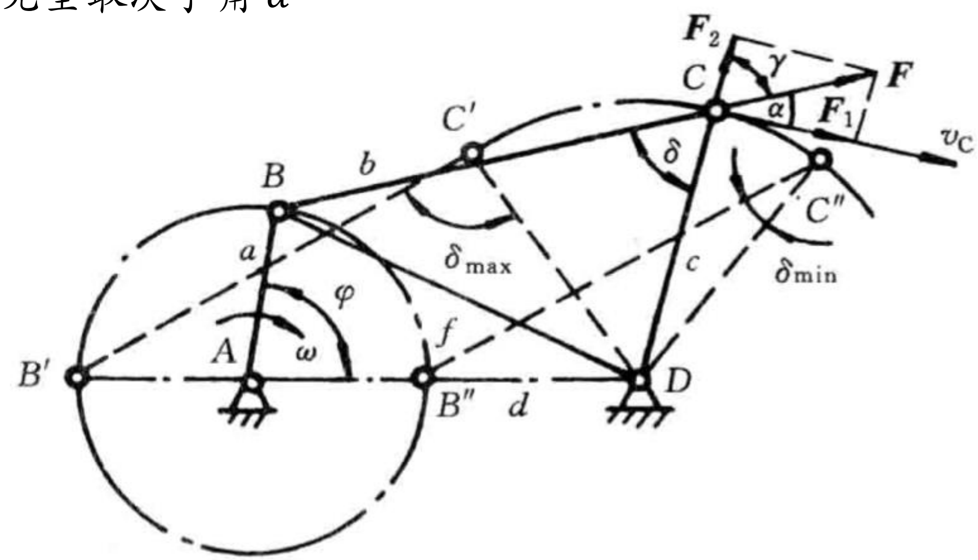
- 在不计摩擦力、惯性力和重力的条件下，机构中驱使输出构件运动的力的方向线与输出构件上受力点的速度方向间所夹的锐角（余角 γ 为传动角：$\gamma = 90° - \alpha$）
  - 主动构件AB上的驱动力通过连杆BC传给输出构件CD的力 F 是沿BC方向
  - 设力 F 与速度 v_c 方向之间所夹的锐角为 α
  - $F_1 = F\cos\alpha$：对从动件产生有效转动力矩
  - $F_2 = F\sin\alpha$：只增加摩擦力矩，无助于输出构件的转动
- 为使机构传力效果良好，显然应使 F_1 的值愈大愈好
  - 理想情况是 $\alpha = 0°$，最坏的情况是 $\alpha = 90°$
  - 在力 F 一定的条件下，F_1、F_2 的大小完全取决于角 α

为了提高机械的传动效率，对于一些承受短暂高峰载荷的机构，应使其在具有最小传动角的位置时，刚好处于工作阻力较小（或等于零）的空回行程中

为了保证机构的传力效果，应限制机构的压力角的最大值 α_{max} 或传动角的最小值 γ_{min} 在某一范围内

$$\gamma_{min} \geq [\gamma] \text{ 或 } \alpha_{max} \leq [\alpha]$$

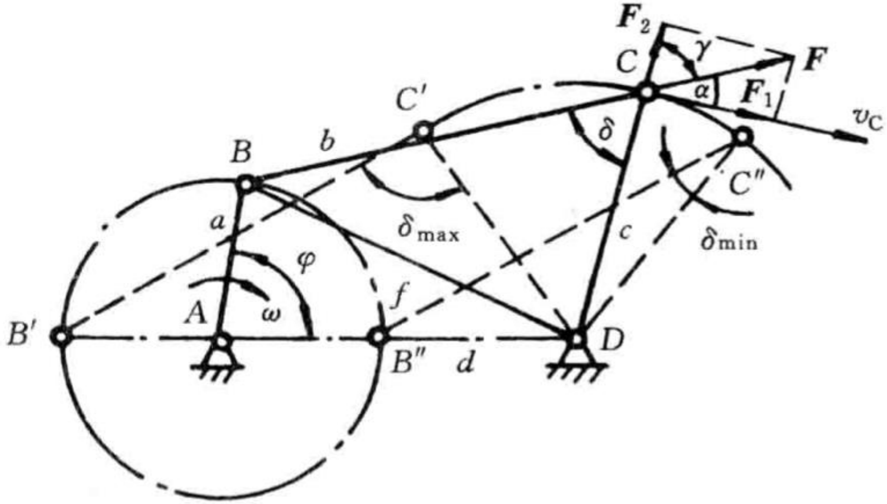
- 一般机械：$[\gamma] = 30°\sim60°$
- 高速和大功率机械：$[\gamma]$ 应取较大值



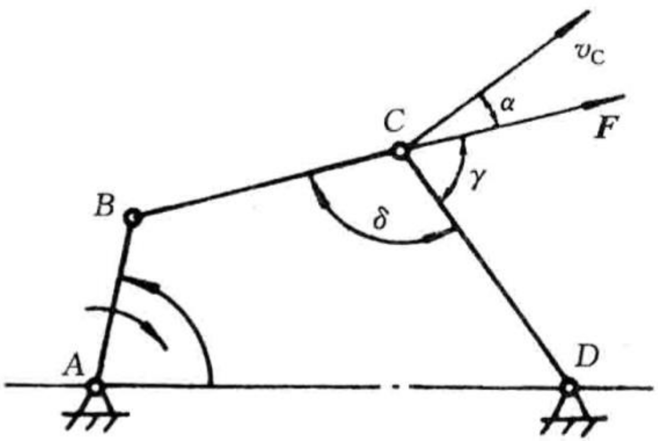
最小传动角的确定

- 对已设计好的平面四杆机构，应校核其压力角或传动角，以确定该机构的传力特性。为此，必须找到机构在一个运动循环中出现最小传动角（或最大压力角）的位置及大小

- 若 δ 为锐角，则 $\gamma=\delta$；若 δ 为钝角，则 $\gamma=180°-\delta$
  - 故当 δ 在最小值或最大值的位置时，有可能出现传动角的最小值



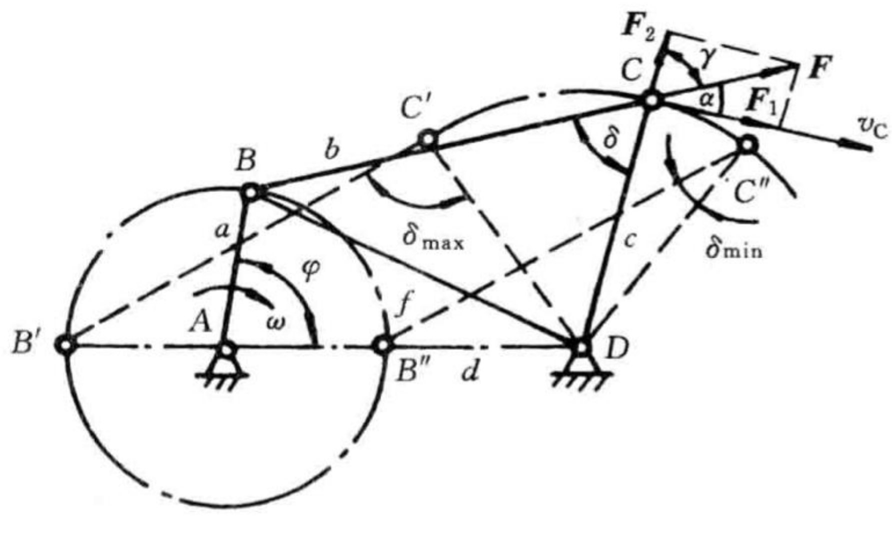
(a)



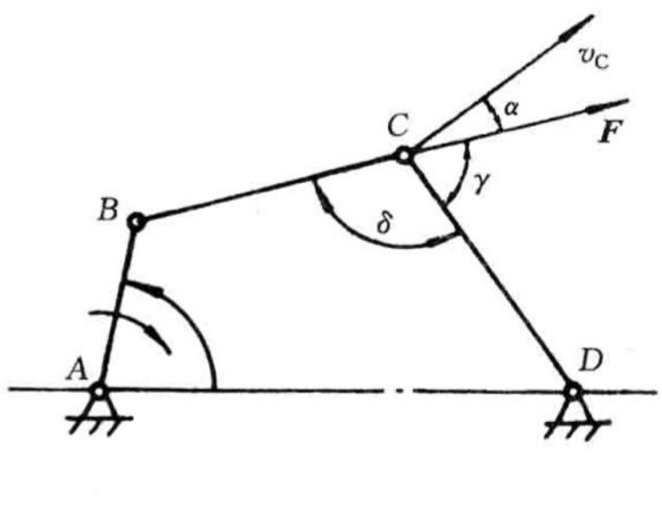
(b)

最小传动角的确定

- $f^2 = a^2 + d^2 - 2ad\cos\varphi$，$f^2 = b^2 + c^2 - 2bc\cos\delta$
- $\delta = \arccos\frac{b^2+c^2-a^2-d^2+2ad\cos\varphi}{2bc}$
  - (1) 当 $\varphi = 0°$，即AB与机架AD重叠共线时，得 $\delta_{min} = \arccos\frac{b^2+c^2-(d-a)^2}{2bc}$
  - (2) 当 $\varphi = 180°$，即AB与机架AD拉直共线时，得 $\delta_{max} = \arccos\frac{b^2+c^2-(d+a)^2}{2bc}$
- 所以： $\gamma_{min} = \min\{\delta_{min}, 180° - \delta_{max}\}$



(a)



(b)

最小传动角的确定

- 对于偏置曲柄滑块机构
  - 当曲柄为主动件、滑块为从动件时，$\cos\gamma = \frac{a\sin\varphi + e}{b}$
  - 当 $\varphi = 90°$ 时，有 $\gamma_{min} = \arccos\frac{a+e}{b}$
  - 曲柄滑块机构可视为由曲柄摇杆机构演化而成
    - 所以，曲柄与机架的共线位置应为曲柄垂直于滑块导路线的位置
    - 故 γ_{min} 必然出现在 $\varphi = 90°$ 时的位置

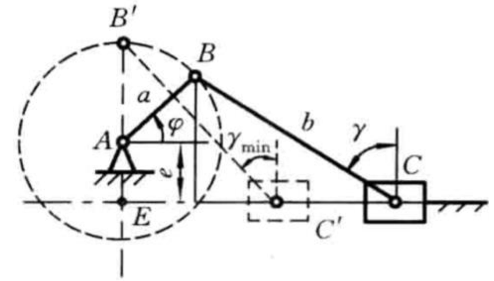
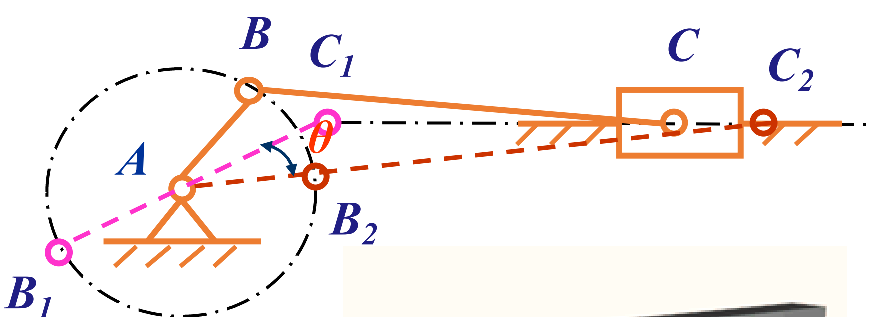
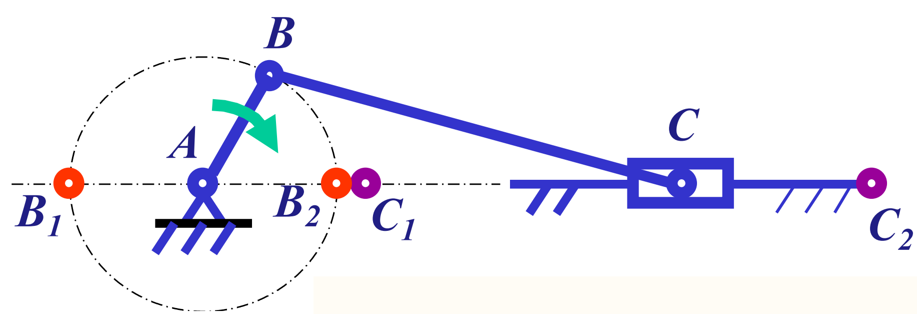
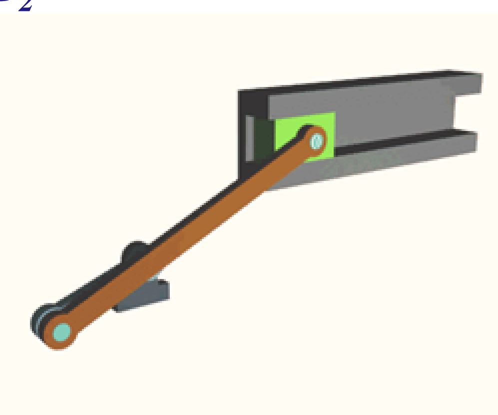
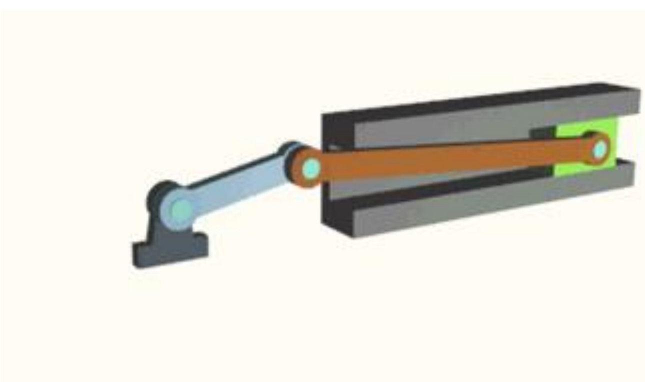


图 2-20 偏置曲柄滑块机构的传动角









最小传动角的确定

- 为使机构具有最小传动角的瞬时位置能处于机构的非工作行程中
  - 注意滑块的偏置方位、工作行程方向与曲柄转向的正确配合
    - 例如，当滑块偏于曲柄回转中心的下方，且滑块向右运动为工作行程，则曲柄的转向应该是逆时针的
    - 反之，若滑块向左运动为工作行程，则曲柄的转向应该是顺时针的，这样也可以同时保证输出件滑块具有良好的传力性能
  - 在设计偏置曲柄滑块机构时，可采用下述方法判别偏置方位是否合理：

过曲柄回转中心A作滑块上铰链中心C的移动方位线的垂线 → 将其垂足E视为曲柄上的一点 → 则当E与滑块的工作行程方向一致时，说明主动件曲柄的转向以及滑块的偏置方位选择是正确的；否则，应重新设计 → 可利用 $\gamma_{min} = \arccos\frac{a+e}{b}$ 判别偏置方位合理性

机构的“死点”位置

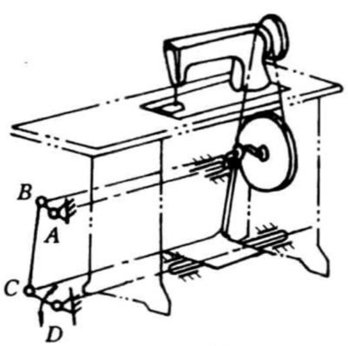
- 无论给机构主动件上的驱动力或驱动力矩有多大，均不能使机构运动，这个位置称为“死点”位置

不利的“死点”例子

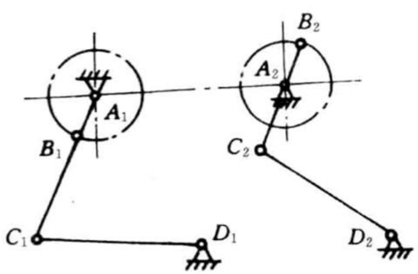
- 缝纫机主动件是摇杆（踏板）CD，输出件是曲柄AB
- 当曲柄与连杆共线时，$\gamma=0^{\circ}$，主动件摇杆给输出件曲柄的力将沿着曲柄的方向
- 不能产生使曲柄转动的有效力矩，当然也就无法驱使机构运动

有利的“死点”例子

- 在工程实践中，不少场合要利用“死点”位置来满足一定的工作要求
- 如图所示的钻床上夹紧工件的快速夹具，就是利用“死点”位置夹紧工件的一个例子
- 如图所示的飞机起落架机构也是利用“死点”位置进行工作的一个例子



(a) 缝纫机



(b) “死点”位置

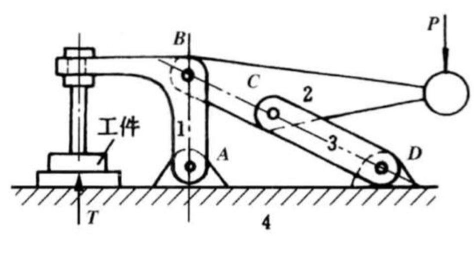


图 2-23 利用“死点”位置夹紧工件

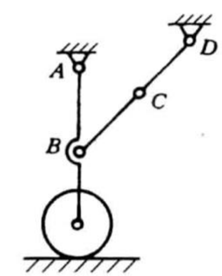


图 2-24 飞机起落架机构

谢谢~

宋超阳
songcy@ieee.org