ME311 Quiz 2/3

本试卷共（2）大题，满分（100）分，请用中文或英文作答，将答案单独写在答题纸上，在答题纸右上角标注姓名学号。

There are 2 problems in total for 100 points. Write answers in a separate sheet in Chinese or English. Identify your name and student number on the top right corner of your answer sheets.

1. （45 分）一对标准安装的渐开线标准直齿圆柱齿轮外啮合传动，已知: a=100mm，z_1=20, z_2=30, α= 20°, d_{a1}= 88mm。
   For a pair of standard involute cylindrical spur gears, a=100mm, z_1=20, z_2=30, α= 20°, d_{a1}= 88mm.

(1) 试计算下列几何尺寸:
Please calculate

(i) （5 分）齿轮的模数 m ;

the module, m;

(ii)（6 分）两轮的分度圆直径 d_1, d_2 ;

the reference diameter of the 2 gears, d_1, d_2;

(iii)（6 分）两轮的齿根圆直径 d_{f1}, d_{f2};

the dedendum circle diameter, d_{f1}, d_{f2};

(iv)（6 分）两轮的基圆直径 d_{b1}, d_{b2} ;

the base circle diameter, d_{b1}, d_{b2};

(v)（5 分）顶隙 c。

the clearance, c.

(2) 若安装中心距增至 a'= 102mm，试问:
If the centre distance becomes a’=102mm,

(i)（5 分）上述各值有无变化，如有，应为多少？

Is there any change in the above values? If yes, calculate the new value.

(ii)（12 分）两轮的节圆半径 r_1'、r_2'和啮合角 α'为多少?

What are the pitch circle radii r_1', r_2', and the working pressure angle α'?

答案：

渐开线齿轮传动具有可分性，中心距加大后其传动比不变，但两节圆分别大于分度圆，啮合角大于压力角。此时实际中心距 a'与啮合角α'的关系为：$a'cos\alpha' = acos\alpha$.

For a pair of involute gears, the transmission ratio will not change when the center distance grows, but the pitch circles will be larger than the reference circles, and the working pressure angle will also increase. The relationship between the real center distance, a', and the working pressure angle, α', is: $a'cos\alpha' = acos\alpha$.

(1) 几何尺寸计算

(i)（5 分）模数 m:

The module, m:

$$m = 2a/(z_1 + z_2) = 2 \times 100/(20 + 30)\ \text{mm} = 4\ \text{mm}$$

(ii)（6 分）分度圆直径 d_1, d_2 ;

the reference diameter of the 2 gears, d_1, d_2;

$$d_1 = mz_1 = 4 \times 20\ \text{mm} = 80\ \text{mm}$$
$$d_2 = mz_2 = 4 \times 30\ \text{mm} = 120\ \text{mm}$$

(iii)（6 分）两轮的齿根圆直径 d_{f1}, d_{f2};

the dedendum circle diameter, d_{f1}, d_{f2};

$$d_{f1} = d_1 - 2h_f = [80 - 2 \times 4 \times (1 + 0.25)]\ \text{mm} = 70\ \text{mm}$$
$$d_{f2} = d_2 - 2h_f = [120 - 2 \times 4 \times (1 + 0.25)]\ \text{mm} = 110\ \text{mm}$$

（其中：$h_a^* = (d_{a1} - d_1)/(2m) = 1$，$c^* = 0.25$）

(iv)（6 分）两轮的基圆直径 d_{b1}, d_{b2} ;

the base circle diameter, d_{b1}, d_{b2};

$$d_{b1} = d_1 \cos\alpha = 80 \times \cos 20^\circ\ \text{mm} = 75.175\ \text{mm}$$
$$d_{b2} = d_2 \cos\alpha = 120 \times \cos 20^\circ\ \text{mm} = 112.763\ \text{mm}$$

(v)（5 分）顶隙 c。

the clearance, c.

$$c = c^* m = 0.25 \times 4 \text{ mm} = 1 \text{ mm}$$

(2) 安装中心距增加至 a'= 102mm。

When the center distance changes to 102mm.

(i)（5 分）

上述各值只有顶隙一项有变化（1 分），其变化后的值为

Only the clearance will change, and the new value is

$$c = (1+2)mm = 3mm。 \qquad (4\text{ 分})$$

(ii)（12 分）

（4 分）啮合角：

The working pressure angle:

$$\alpha' = \arccos(a\cos\alpha/a') = \arccos(100 \times \cos 20°/102) = 22.888°$$

（8 分）节圆半径 r_1'、r_2'

The pitch circle diameters, r_1', r_2':

$$r_1' = r_{b1}/\cos\alpha' = 40.8 \text{ mm}$$
$$r_2' = r_{b2}/\cos\alpha' = 61.2 \text{ mm}$$

2.（55 分）如图 1 所示为某传动装置示意图，已知减速器输出轴上的功率 P=11kW，转速 n=210 r/min，单向旋转。大齿轮受力；圆周力$F_t = 2619N$,径向力$F_\tau = 982N$，轴向力$F_a = 653N$。大齿轮分度圆直径$d_2 = 382\text{mm}$，轮毂宽度 B=80mm，试设计该输出轴，并按弯扭合成强度校核轴。

A schematic diagram of a reducer is shown in the figure. It is known that the power on the output shaft of the reducer is P=11kW, speed n=210 r/min (unidirectional rotation). The large gear is subjected to forces. The circumferential force $F_t = 2619N$, the radial force $F_\tau = 982N$, and the axial force $F_a = 653N$. The diameter of the reference circle of the large gear $d_2 = 382\text{mm}$ and wheel hub width B=80mm. Try to design this output shaft and check the strength of this shaft by resulting bending and torsion strength.

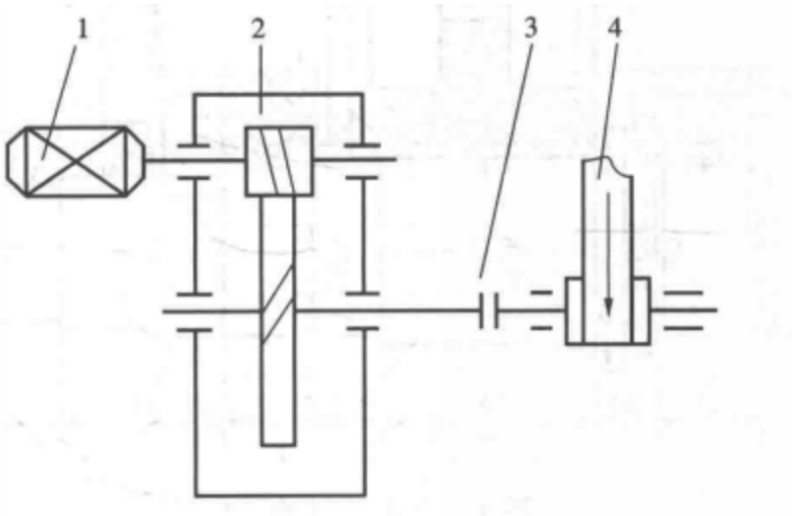

图 1 Figure 1

1-电动机 2-减速器 3-联轴器 4-输送器

1-electric motor 2-reducer 3-coupling 4-conveyor

答案：

以输出轴为研究对象，该轴既受弯矩又传递转矩，为转轴。因无特殊要求，则按照一般转轴标准设计，设计计算过程如下。

The output shaft is subject to bending moment and transmits torque, so it is a rotating shaft. Since there are no special requirements, the design is based on general shaft standards. The design calculation process is as follows.

（1）（5 分）选择轴的材料，确定许用应力。

Select the material of the shaft and determine the allowable stress.

该轴无特殊要求，选用 45 钢，调质处理。查表可得$\sigma_b = 650MPa, [\sigma_{-1b}] = 60MPa$。

There are no special requirements for this shaft, so choose tempering treated 1045 carbon steel as the material. Look up the table to get $\sigma_b = 650MPa, [\sigma_{-1b}] = 60MPa$.

（2）（21 分）轴的结构设计。Design of the structure of the shaft.

①初步估计最小直径。按照扭转强度估算输出轴的最小直径。查表，可取$C = 110$，则

Estimate the minimum diameter of the output shaft based on torsional strength. Looking up the table, we can pick $C = 110$, then

$$d \geqslant C\sqrt[3]{\frac{P}{n}} = 110\sqrt[3]{\frac{11}{210}}\ \text{mm} = 41.2\ \text{mm}$$ （2 分）

考虑到轴上开有键槽，故将轴的直径增加 4%，

Considering that there are keyways on the shaft, increase the diameter by 4%.

$$d = 41.2 \times (1 + 4\%)mm = 42.9mm$$ （2 分）

此段轴的直径和长度应与联轴器相符，所以选用 LT7 型弹性套柱销联轴器，其轴孔直径为 45mm。 （1 分）

The diameter and length of the shaft in this section should be consistent with the coupling, so the LT7 elastic sleeve pin coupling is selected, and its shaft hole diameter is 45mm.

②确定轴上零件安装方式。单级减速器，齿轮布置在箱体中央，两轴承对称布置，轴的外伸端安装联轴器。轴做成阶梯状，齿轮、套筒、右端轴承及轴承盖和联轴器均从右端装入，而左端轴承和轴承盖从左端装入。（1 分）

Determine how parts are installed on the shaft. Since this is a single-stage reducer, the gear is arranged in the center of the box, the two bearings are arranged symmetrically, and the coupling is installed on the extended end of the shaft. The shaft is made into a stepped shape, the gear, sleeve, right-end bearing, bearing cap and coupling are all installed from the right end, while the left-end bearing and bearing cap are installed from the left end.

③ 确定轴各段直径。如图 14-2 所示，外伸段直径 $d_①$ = 45 mm；考虑右端联轴器的定位需要，轴段② 直径 $d_②$ = $d_①$ + (5～10) mm，取 $d_②$ = 52 mm；轴段③、⑦ 均与轴承配合，此两处轴承选用 6311 型，则 $d_③$ = $d_⑦$ = 55 mm；考虑齿轮的装拆，$d_④$ = $d_③$ + (1～3) mm，取 $d_④$ = 58 mm；由齿轮的定位要求，$d_⑤$ = $d_④$ + (5～10) mm，取 $d_⑤$ = 68 mm；轴段⑥ 、⑦ 间的轴肩用于左滚动轴承的定位，由滚动轴承 6311 查得 $d_⑥$ = 65 mm。

③ Determine the diameter of each segment of the shaft. As shown in fig.2, $d_1 = 45mm$. Considering the positioning needs of the right end coupling, the diameter of segment 2, $d_2 = d_1 + (5\sim10)mm$, take $d_2 = 52mm$. Both segment 3 and 7 need to match the bearings. If we

use type 6311 bearings, then $d_3 = d_7 = 55mm$. Considering the assembly of the gears, $d_4 = d_3 + (1\sim3)mm$. Take $d_4 = 58mm$. According to the positioning requirements of the gears, $d_5 = d_4 + (5\sim10)mm$. Take $d_5 = 68mm$. The shoulder between shaft segments 6 and 7 is used to position the left rolling bearing, so according to the type 6311 bearing, it can be looked up that $d_6 = 65mm$. (每段轴的直径各 1 分，共 7 分)

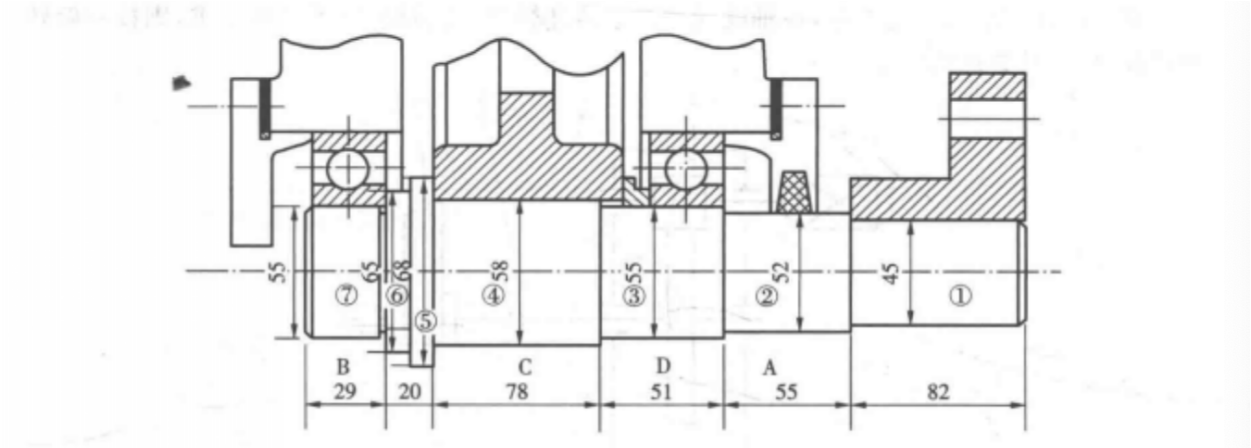

图 2 输出轴的结构图

Figure 2. the structure of the output shaft

④ 确定轴各段长度。齿轮轮毂宽度 B=80 mm，为保证齿轮轴向固定可靠，则取轴段④的长度 $L_④$=(80−2) mm=78 mm；查得轴承 6311 的宽度为 29 mm，故 $L_⑦$=29 mm；考虑齿轮两端面、轴承端面应与箱体内壁保持一定距离，故取 $L_⑤$=10 mm，$L_⑥$=10 mm，套筒长度为 20 mm，则 $L_③$=(2+20+29) mm=51 mm。根据箱体结构要求和联轴器距箱体外壁要有一定距离的要求，取 $L_②$=55 mm；与联轴器配合的轴段长 $L_①$=(84−2) mm=82 mm。

④ Determine the length of each segment of the shaft. Since the wheel hub width B=80mm, In order to ensure the axial fixation of the gear, take $L_4 = (80 - 2)\ mm = 78\ mm$. The width of 6311 bearing is 29 *mm*, so take $L_7 = 29\ mm$. Considering that the end faces of the gears and the end face of the bearing should be kept at a certain distance from the box, take $L_5 = 10\ mm$, $L_6 = 10\ mm$. The length of the sleeve is 20 *mm*, so $L_3 = (2 + 20 + 29)mm = 51mm$. According to the structural requirements of the box and the requirement that the coupling needs to maintain a certain distance from the outer surface of the box, take $L_2 = 55\ mm$. Segment 1 matches the coupling, $L_1 = (84 - 2)mm = 82mm$. （每轴段长度各 1 分，共 7 分）

根据以上的结构设计，得到输出轴的结构设计草图如图 2 所示。并且得到轴的两支点跨度 $L = 149mm$，右支点到联轴器的距离$K = 110.5mm$。（1 分）

According to the design above, the design draft of the output shaft is as shown in fig. 2. The span of the two fulcrums of the shaft is L=149*mm*, and the distance from the right fulcrum to the coupling is K=110.5*mm*.

（3）（29 分）按弯扭合成强度校核轴。

Check the shaft according to the bending and torsion strength.

①绘制轴的受力简图（如图 3(a)所示）。 （3 分）

Draw the force diagram of the shaft, as shown in fig.3(a).

②作垂直面内轴的受力简图，求支反力，并作垂直面内的弯矩图（见图 3(b)）（作图 2 分，计算 4 分）

Draw the force diagram of the shaft in the vertical plane, find the support reaction force, and draw the bending moment diagram in the vertical plane (as shown in fig.3(b))

$$F_{BV}=\frac{F_r\times\frac{L}{2}-F_a\times\frac{d_2}{2}}{L}=\frac{982\times\frac{149}{2}-653\times\frac{382}{2}}{149}\ \text{N}=-346\ \text{N}(\text{与假设方向相反})$$

$$F_{AV}=F_r-F_{BV}=(982+346)\ \text{N}=1\ 328\ \text{N}$$

$$M'_{CV}=F_{BV}\times\frac{L}{2}=\left(346\times\frac{149}{2}\right)\ \text{N}\cdot\text{mm}=25\ 777\ \text{N}\cdot\text{mm}$$

$$M_{CV}=F_{AV}\times\frac{L}{2}=\left(1\ 328\times\frac{149}{2}\right)\ \text{N}\cdot\text{mm}=98\ 936\ \text{N}\cdot\text{mm}$$

③ 作水平面内轴的受力简图，求支反力，作水平面内的弯矩图（见图 3(c)）（作图 2 分，计算 2 分）

Draw the force diagram of the shaft in the horizontal plane, find the support reaction force, and draw the bending moment diagram in the horizontal plane (as shown in fig.3(c))

$$M_{CH}=F_{AH}\times\frac{L}{2}=1\ 309.5\times\frac{149}{2}\ \text{N}\cdot\text{mm}=97\ 558\ \text{N}\cdot\text{mm}$$

$$F_{BH}=F_{AH}=\frac{F_t}{2}=\frac{2\ 619}{2}\ \text{N}=1\ 309.5\ \text{N}$$

④ 作合成弯矩图（见图 3(d)）。由$M=\sqrt{M_{CH}^2+M_{CV}^2}$得。 （作图 3 分，计算 2 分）

Draw the resultant bending moment diagram, as shown in fig.3(d). Since $=\sqrt{M_{CH}^2+M_{CV}^2}$,

$$M'_C=\sqrt{M_{CH}^2+M'^2_{CV}}=\sqrt{97\ 558^2+25\ 777^2}\ \text{N}\cdot\text{mm}=100\ 906\ \text{N}\cdot\text{mm}$$

$$M_C=\sqrt{M_{CH}^2+M_{CV}^2}=\sqrt{97\ 558^2+98\ 936^2}\ \text{N}\cdot\text{mm}=138\ 946\ \text{N}\cdot\text{mm}$$

⑤ 作轴的扭矩图（见图 3(e)）。 （作图 2 分，计算 1 分）

Draw the torque diagram, as shown in fig.3(e).

$$T=F_t\times\frac{d_2}{2}=\left(2\ 619\times\frac{382}{2}\right)\ \text{N}\cdot\text{mm}=500\ 229\ \text{N}\cdot\text{mm}$$

⑥ 画当量弯矩图，确定危险截面。由当量弯矩图和轴的结构图可知，C 截面和 D 截面（3，4 轴段间的阶梯处）都有可能是危险截面，其当量弯矩为

Draw the equivalent bending moment diagram to determine the dangerous cross section. It can be seen from the equivalent bending moment diagram and the structural diagram of the shaft that both the C section and the D section (the step between the 3rd and 4th axis segments) may be dangerous sections, and their equivalent bending moment are

$$M_e = \sqrt{M^2 + (\alpha T)^2}$$

设轴的扭转切应力为脉动循环应力，取$\alpha = 0.6$，

Suppose the torsional shear stress of the shaft is the pulsating cyclic stress, and take α=0.6.

对于 C 截面，

For cross section C,

$$M_{Ce} = \sqrt{M_C^2 + (\alpha T)^2} = \sqrt{138\ 946^2 + (0.6 \times 500\ 229)^2}\ \text{N} \cdot \text{mm} = 330\ 739\ \text{N} \cdot \text{mm}$$

对于 D 截面，

For cross section D,

$$M_{DV} = F_{AV} \times \left(51 - \frac{29}{2}\right) = (1\ 328 \times 36.5)\ \text{N} \cdot \text{mm} = 48\ 472\ \text{N} \cdot \text{mm}$$

$$M_{DH} = F_{AH} \times \left(51 - \frac{29}{2}\right) = (1\ 309.5 \times 36.5)\ \text{N} \cdot \text{mm} = 47\ 797\ \text{N} \cdot \text{mm}$$

$$M_{DH} = \sqrt{M_{DV}^2 + M_{DH}^2} = \sqrt{48\ 472^2 + 47\ 797^2}\ \text{N} \cdot \text{mm} = 68\ 074\ \text{N} \cdot \text{mm}$$

$$M_{De} = \sqrt{M_D^2 + (\alpha T)^2} = \sqrt{68\ 074^2 + (0.6 \times 500\ 229)^2}\ \text{N} \cdot \text{mm} = 307\ 761\ \text{N} \cdot \text{mm}$$

作图如图 3(f)。（作图 2 分，计算 3 分）

⑦ 校核危险截面的强度。（计算 2 分，结论 1 分）

Check the strength of the dangerous cross section.

对于 C 截面

For cross section C,

$$\sigma_{Ce} = \frac{M_{Ce}}{0.1 d_{④}^3} = \frac{330\ 739}{0.1 \times 58^3}\ \text{MPa} = 16.95\ \text{MPa} < [\sigma_{-1b}]$$

对于 D 截面

For cross section D.

$$\sigma_{De}=\frac{M_{De}}{0.1d_{③}^{3}}=\frac{307\ 761}{0.1\times55^{3}}\ \text{MPa}=18.5\ \text{MPa}<[\sigma_{-1b}]$$

故轴设计合格，满足强度要求。

Therefore, the shaft design is qualified, since the shaft meets the strength requirements.

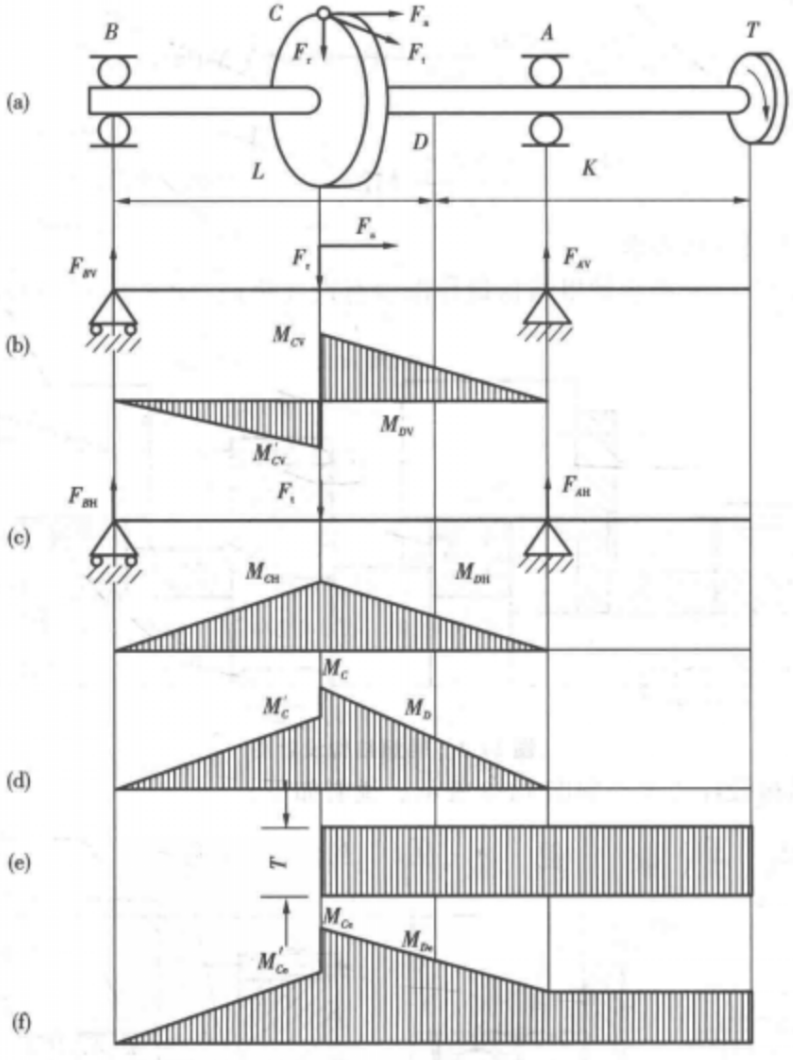

图 3 轴的受力分析及内力图

Figure 3. Force analysis and internal force diagram of the shaft